

## Research Article

# The Nature of Uncertainty in Historical Geographic Information

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### Abstract

While the presence of uncertainty in the geometric and attribute aspects of geographic information is well known, it is also present in temporal information. In spatiotemporal GIS databases and other formal representations, uncertainty in all three aspects of geography (space, time, and theme) must often be modeled, but a good data model must first be based on a sound theoretical understanding of spatiotemporal uncertainty. The nature of both uncertainty inherent in a phenomenon (often termed indeterminacy) and uncertainty in assertions of that phenomenon can be better understood through the *Uncertain Temporal Entity Model*, which characterizes the cause, type, and form of uncertainties in the spatial, temporal, and attribute aspects of geographic information. These uncertainties are the result of complexities and problems in two processes: the process of conceptualization, by which humans make sense of an infinitely complex reality, and measurement, by which we create formal representations (e.g. GIS) of those conceptual models of reality. Based on this framework, the nature and form of uncertainty is remarkably consistent across various situations, and is approximately equivalent in the three aspects, which will enable consistent solutions for representation and processing of spatiotemporal data.

## 1 Introduction

There is a considerable amount of information that has both geographic and historical significance (herein termed *geo-historical* information), including subjects such as the ebb and flow of empires, the changing structure of cities, and the movements of individuals and peoples. Detailed information about these subjects is useful in a variety

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of applications, especially in education and research in history, geography, and other disciplines.

For example, information about historical local administrative units can be used in applications such as land title searches, demographics, genealogy, legal research, and studying the history of settlement and politics. Several projects have tried to capture this information in the US (Earle et al. 1999, Long 1993), the UK (Southall et al. 2000), and other countries. The research presented here was motivated by a project to map the historical county boundaries of Utah, which have changed frequently due to increased settlement, political maneuvering, and increasing knowledge of local geography. Thus, any useful system for modeling historical county boundaries will need to model not only the spatial extent of the counties, but their changes over time.

Database management systems and geographic information systems are important tools for storing, analyzing, and visualizing such large, complex collections of information. However, turnkey software systems for modeling this type of information (with its spatial and temporal aspects) are not generally available. The extensive research to date in temporal GIS and spatiotemporal databases (Al-Taha et al. 1994) has focused attention on these two areas. Early insights into the role of time in geography (Wright 1955, Berry 1964, Hägerstrand 1970) have been further developed into theories of geographic space, theme, and time (e.g. Sinton 1978, Frank et al. 1992, Egenhofer and Golledge 1998, Yattaw 1999). Based on these ideas, theoretical and working data models have been developed to manage spatiotemporal information (e.g. Langran 1992, Peuquet and Qian 1996, Vrana 1990, Gregory 2002). Vrana (1990) and Gregory (2002) are especially relevant, since they discuss historical administrative geography (although this paper does not focus on data modeling). These theories and models are vastly different from one another in scope, terminology, and structure. While some variation is to be expected as concepts are tailored to a particular application, communication between theories and models would be aided if there were some common ground.

Another obstacle to the effective use of geo-historical information is the presence of *uncertainty*. Longley et al. (2001, 124) define uncertainty in a broad sense (although not in a single sentence) as the acknowledgment and consideration of imperfections in information. That is, our representations of reality are not exactly the same as reality itself (often for good reason), and we need to cope with that fact. Some have used the term in a narrow sense, essentially the *assertion uncertainty* discussed later in this paper, but the broad sense seems to have gained general acceptance (probably due to the lack of any other broad term), and will be used herein.

Considerable research has been conducted on the management of uncertainty in GIS and other information systems. Since the early developments presented by Goodchild and Gopal (1989), many advances have been made toward the five goals identified by Veregin (1989, 4): (1) reducing the amount of error and uncertainty; (2) managing information about uncertainty within the database (e.g. Faiz and Boursier 1996); (3) measuring and managing the propagation of uncertainty by GIS operations (e.g. Heuvelink 1998); (4) measuring or estimating the error in geographic data (e.g. Edwards and Lowell 1996); and (5) understanding the sources and causes of uncertainty in geographic information.

According to Veregin (1989), the adequate solution of the first four goals is dependent on the last: a sound theoretical understanding of error and uncertainty. Goodchild (1988, p. 44) reiterated the need for this knowledge: 'Effective solutions are

likely to require much better understanding of the processes which create spatial variation than we currently possess.' Unfortunately, not as much effort has been expended in this task as in the higher-level tasks, but some elements of a 'theory of uncertainty' have been developed, which will be discussed in further detail later. This work builds on these earlier developments.

Unfortunately, the interactions between the temporal nature of geographic information and the uncertainty in that information have not been adequately explored. Unwin (1995) recognizes the need for an uncertainty-savvy temporal GIS, but does not develop such a system. Peuquet (2001) briefly discusses the presence of uncertainty in spatiotemporal information, even noticing some similarity between spatial and temporal uncertainty (which will be further explored in this paper). Pfoser and Tryfona (2000) discuss uncertainty and indeterminacy in spatiotemporal databases, proposing the use of probability and fuzzy sets in spatiotemporal databases, but develop neither a theory of uncertainty, nor a practical implementation.

This paper addresses this need by developing a model of the nature of uncertainty, specifically in representations of the thematic, spatial, and temporal aspects of geo-historical phenomena, called the Uncertain Temporal Entity Model (UTEM). First, a basic model of geographic information is developed by bridging the various concepts of geographic space, time, and existence. It is then extended to form the UTEM, based on observations of the ontological and epistemological nature of geographic and historical information. Such a framework must help the information gatherer isolate the nature of specific uncertainties, and must support a sound strategy for modeling this information in GIS. The first criterion is partially tested by applying the model to several uncertain situations in the history of Utah counties. A specific digital implementation of this theoretical model is being developed, but is not discussed herein.

## 2 A General Model of Geo-Historical Information

A model of geo-historical uncertainty must be based on a general understanding of the nature of information that encompasses all three of the aspects of geographic phenomena: space, time, and theme. Several general models of the three-faceted nature of geography have been proposed, including the geographic matrix of Berry (1964), the measurement framework of Sinton (1978), the time geography of Hägerstrand (1970), and the TRIAD model of Peuquet (1994), which was extended by Mennis et al. (2000). One common thread in these approaches is a high degree of symmetry in how geographic phenomena are manifest in each of the three aspects (which might be collectively called the *geographic universe*). Space, time, and theme are not exactly symmetrical—time and attributes are not just extra spatial dimensions—but their form is similar enough to be useful.

Because uncertainty is a human-induced phenomenon, this model is based on the cognitive structure of the geographical world (ontological tier #2 of Frank 2001), rather than its metaphysical structure (Frank's tier #0). The metaphysics of geographic information has been hotly debated (one symptom being the field/object dichotomy), and are not likely to ever be resolved (Raper 2000). There seems to be much more consensus (although incomplete understanding) on the basic structure of the cognitive world, even between the realists, experiential realists, and social constructivists that

seem to make up most of the GIScience community. The only metaphysical stand taken herein is that space, time, and matter (the basis for theme) are 'real'; that is, they exist independently of any observer (their measurement systems do not, of course). Beyond that, the model accepts the possibility of more complex structure, but does not require it. We will thus use the term *phenomena* to mean whatever is out there to be observed.

### 2.1 Entities and Their Manifestations

The common-sense geographic world appears to be richer than just measurements in the three domains. Most importantly, we seem to structure our world primarily around objects or *entities* (Frank 2001, 670), distinct geographic phenomena of any type, size, or form. Examples include a person, a country, a culture region, or even the atmosphere. As opposed to the three 'anonymous' domains, entities have significant meaning to people. This gives each entity an *identity* independent from its particular place in the geographic universe (as used by Hornsby and Egenhofer 2000), even if they were originally created from observations of space, time, and theme (although some consider it merely a special case of theme).

Although identity is a separate issue, each entity is manifest in the geographic universe (Mennis et al. 2000, 508). That is, the entity occurs at particular places (its *location*), at particular times (its *lifespan*), and with particular attribute values (its *description*). Collectively, these three manifestations will be called the *extent* of the entity. The extent is not divided into space, time, and theme quite so simply, because multiple aspects interact in each manifestation. The location and description can each vary over time (within the overall lifespan); that is, different points in space and attribute values are valid parts of the extent at various times. Furthermore, many (but probably not all) *fields* can be considered attributes of entities that vary over space (within the overall location) as well as time, such as the density of a population or the temperature of a lake (Wright 1955).

### 2.2 Representations

Regardless of one's metaphysical stance, it is clear that the cognitive, visual, and digital models we use are never perfect duplicates of reality (which is unmanageable in its full complexity), but are rough approximations derived through a very involved process of *representation*. The details of this process vary from one situation to another, but it is generally generalized into two major steps, *conceptual* and *physical modeling*. This distinction is useful for understanding uncertainty; in fact, Longley et al. (2001, 124) use it to frame a description of uncertainty in GIS that is essentially a simpler form of the model presented herein.

The first modeling process is *conceptualization*, in which one or more real phenomena (the *referents*) are observed and organized mentally to form a *conceptual entity*, with its *ideal* extent. This conceptual entity has one of three ontological natures: (1) a direct correspondence to a single real entity (the *bona fide* entity of Smith 1995), such as a person (if there are any, a person seems like a good candidate); (2) an artificial entity created publicly (the *fiat* entity of Smith), such as a country; or (3) a purely mental construct based on patterns in a much more complex set of real phenomena by cognitive processes such as aggregation and categorization (the *motivated* phenomena of Plewe 1997, 24), such as a storm or a culture region.

The second step produces useful, external information (such as a GIS) through the *measurement* process, using methods such as observation, interpretation, generalization, organization, classification, and encoding. This produces an *asserted* extent for the entity, which should match the ideal extent.

### 2.3 Measurement and Control

Both of these processes fit within the measurement framework of Sinton (1978), in which one of the three aspects is measured based on controlled (i.e. selected by the observer) and fixed values of the other two. However, the entity-manifestation model enables some important extensions:

- Entity identity is available as a fourth variable, usually as control.
- The distinction between the fixed and control variables is relaxed. While some measurement devices and data models really do have fixed variables, they usually have multiple control variables (but are not varied at the same time).
- Each manifestation is a separate measurement process. That is, the name of a town's mayor and its location may depend on time, but not on the lifespan, or each other. Thus, all three can be measured simultaneously, using different measurement tools.

Therefore, a *datum* of conceptual or asserted information (whether in space, time, or theme) can be generalized as an *information function* of values of one or more control variables, called *parameters*:

$$d = f(p_1, p_2, \dots, p_n) \quad (1)$$

For example, the population of a town entity would be determined by the identity of the particular town and time. The function is constructed by the conceptualization and/or measurement processes, which in this case might be quite complex, taking into account the meaning of the term 'town,' and the changing locations of people and urban infrastructure. For the lifespan and location, the datum may be a set of points (e.g. the entire areal extent of a town), while most attributes have only a single value.

Although time is almost always a control variable (largely due to its natural immotility) in the conceptualization and measurement processes, it is often not controlled directly. Often, the times selected for the taking of measurements at significant, meaningful moments, whether they are samples of continuous change (e.g. a census of population) or points of sudden change (e.g. the ignition of a forest fire). These *events* (Peuquet and Duan 1995), with an independent identity just like entities, are then the actual control. The exact time at which each event occurred is itself a measurement with the event identity as control.

Thus, the conceptual framework does not have perfect symmetry; space, time, and theme occupy different (if somewhat similar) parts in the creation and manifestation of entities. The symmetry lies in the fact that each can function, in its turn, as a control parameter or as a measured datum; these roles are more crucial to understanding uncertainty than the particulars of which aspect is which in a particular measurement system.

The conceptualization and measurement processes are both unavoidably imperfect: the resultant entities are simplifications of the referents (i.e. detail has been lost), and asserted or ideal values differ from their real counterparts. The necessary

existence of these discrepancies means that any asserted extent has some degree of uncertainty (although it is often insignificant for a particular application).

Processing and analysis of this information (e.g. GIS overlay) constitute further transformations of the asserted extent, producing new asserted extent sets of the same or new conceptual entities that are almost always more uncertain than the original. In addition, communication results in uncertainty, since the conceptual model of the data creator may not correspond to that assumed by the data user. The uncertainties from these two 'after effects' are discussed by Gottsegen et al. (1999) and Longley et al. (2001, 137), but are not considered herein; this work looks at geographic information *at its best* (i.e. in its original state).

### 3 A Model of Geo-historical Uncertainty

The purpose of this work is to develop an understanding of the nature of uncertainty found in assertions of geo-historical phenomena based on the above framework. While a complete Theory of Geographic Uncertainty is not yet mature, many pieces have been previously developed. Robinson and Frank (1985, 443) present an extensive (although random) list of reasons why spatial data are not entirely accurate. Burrough and McDonnell (1998, 225) list several sources of error, and contexts that can exacerbate or mitigate error. Goodchild (1988, 34) distinguishes between errors in spatial measurement, attribute measurement, and modeling; to this Openshaw (1989, 264) adds several sources similar to Robinson and Frank (1985). Couclelis (1996) discusses several common characteristics of indeterminate geographic phenomena. Fisher (1999) gives probably the clearest explanation to date of the difference between uncertainty due to vagueness and that due to error. Gottsegen et al. (1999) identify the parts of the processes of data development and data query that can produce uncertainty, but do not fully elaborate on how uncertainty arises in these processes.

In nonspatial (i.e. attribute) databases, Motro (1997), Smets (1997) and others differentiate uncertainty and error, as defined above, from *imprecision*, in which broad assertions are made to insure the inclusion of the true value. Both authors also give several reasons why these different types of uncertainty may arise. Worboys (1998) develops a framework to explain imprecision in spatial and attribute information.

One shortcoming of these earlier approaches is that most of them approach the problem from a narrow perspective of subject matter and usage (i.e. the application domain of the researcher), so the resultant models contain only a few elements of uncertainty. In addition, variant terminology has made it difficult to bring the various concepts together. Terms such as *imprecision*, *indeterminacy*, *gradation*, *fuzziness*, *vagueness*, and *indistinctness* are often used with different (and often vague) definitions. Even the term uncertainty itself is problematic, as illustrated by an accidental statement in a recent journal article (the author may want to remain anonymous), 'There are two basic kinds of uncertainty . . . : fuzziness and uncertainty.'

Another difficulty has been that previous lists often group together different aspects of the nature of uncertainty, that play different roles in its understanding and subsequent management. The four aspects that appear most commonly, and are therefore probably the most important, include:

- **Dimensionality of Uncertainty:** in which aspect(s) of an entity (space, time, or property) does the uncertainty manifest itself? Goodchild (1988) and Burrough and

McDonnell (1998, 222) describe the presence of error and uncertainty in the spatial and thematic aspects, while Kahn and Gorry (1977) and Ratcliffe (2000) acknowledged its presence in temporal data. One hypothesis of the UTEM is that uncertainty is similar in each of the three aspects.

- **Cause of Uncertainty:** what happened while creating the information that resulted in the final product being uncertain? There are invariably many steps in each of the representation processes, and problems or complexities in any of them can result in some kind of uncertainty. The variety of possible modeling processes makes this the most difficult of the four aspects to understand conclusively.
- **Type of Uncertainty:** if we say that an assertion is ‘uncertain,’ what exactly is the problem? Several kinds of complexity are usually grouped under the term ‘uncertainty’ in its broadest sense, although they are very different in their nature. Unfortunately, ‘type’ is a frequently confused term (it has been used in the past to refer to any and all of the four aspects listed here), but it seems to fit best here.
- **Form of Uncertainty:** what does the uncertainty look like? Is there a region or interval of possible validity, or is it a single proposition with an uncertain answer? Uncertainty can take a variety of guises, so many different structures are or could be used to model it.

To develop the Uncertain Temporal Entity Model, several situations were evaluated according to these four elements and the process of representation, including the Utah counties described above and others. As uncertainties were found, their forms and sources were investigated and noted.

## 4 Causes and Types of Uncertainty

The two-step representation process and the information function discussed above are valuable tools for understanding these four aspects of uncertainty. According to the epistemological definition of uncertainty given above, Reality itself cannot be uncertain (although it is extremely complex), so uncertainty must be caused by either the conceptualization process or the measurement process. Uncertainty arising in either of these processes will be manifest in the ideal and asserted extent sets, so these can be evaluated to find the possible forms in any or all of the three aspects of the geographic universe.

Each process constructs a set of information functions for each manifestation. Therefore the first place to investigate is this function. Uncertainty in the datum must be the result of a problem in either the function itself or the control parameters. In each section below, the problems resulting from the process creating the function will be considered, then the role of the parameters in uncertainty.

### 4.1 Uncertainty in the Conceptualization Process

Conceptual models are generally simplifications of a reality that is, in all likelihood, infinitely complex. In addition, conceptual models usually impose more order on real phenomena than is probably inherently there. Both processes result in discrepancies. Because the discrepancies arising from the conceptualization process are an unavoidable part of the conceptual model, they can be called *inherent uncertainty*, often termed *indeterminacy* (Burrough and Frank 1996).

#### 4.1.1 Types of Conceptual Entities

The process of building a conceptual entity, and its ideal extent, varies based on the metaphysical nature of the entity itself, whether *bona fide*, fiat, or motivated.

By definition, *bona fide entities* are inherently real. For example, if a person is a *bona fide* entity, then we think of the person as a person. Whether *bona fide* entities are prevalent, rare, or nonexistent in our world is an issue of philosophical debate not considered herein. It is sufficient to say that they may exist, and if so, they cannot be uncertain, because there is no transformation in the conceptualization process. Many *bona fide* entities are very peculiar or complex (e.g. fractal-like coastlines, fields with a high frequency of change, and others discussed by Couclelis (1996)) in ways that require simplification in the measurement process, but it is the asserted extent that is uncertain, not the entity.

A *fiat entity*, such as a country or census enumeration district, owes its existence solely to an official or legal action (e.g. a legislative act or treaty), and may cease to exist due to a similar action. Its lifespan is therefore implied by these actions, although it may or may not be stated clearly. Its spatial extent is usually explicitly specified in the organic action or any subsequent modifying actions (which may be official or *de facto*, such as a war with a moving battle front). The location definition may exist only 'on paper,' composed of explicit relationships to one or more referents, either *reference entities* (e.g. the boundary follows a ridge or a river) or a common *reference system* (e.g. starting on a particular calendar date). Alternatively, it may have been made manifest 'on the ground' in some way (e.g. a surveyed county boundary). Attributes in fiat entities can have three sources: explicit statements in the organic and modifying actions (e.g. county name), subsequent *de jure* or *de facto* activities resulting from these actions (e.g. elections), or natural consequences of the location, lifespan, or other attribute manifestations (e.g. population).

In fiat entities, uncertainty in lifespan, location, or description is generally only present when defined on paper, due to problems in the text. As they are made manifest in reality, uncertainties are generally resolved arbitrarily (e.g. a surveyed, monumented boundary often has legal precedence over a legal description, even if it has mistakes).

*Motivated entities* are purely conceptual phenomena created from more complex phenomena by various processes of simplification (e.g. aggregation, categorization), including things like mountains, forests, and regions (Plewe 1997). The information functions of a motivated entity are based on a definition (often of a broad category of which this is an instance), as does a fiat entity, although it is often, if not always, much more difficult to describe concisely (despite the attempts by lexicographers), as shown by Rosch (1978). This definition (which may more appropriately be termed the *intension*) provides a rule stating the kinds of points that are and are not part of the extent set, based on their locations and attributes (the parameters). For example, the intension of 'mountain' is usually based on patterns of relative elevation. Therefore, the ideal extent of a motivated entity is the theoretical application of the intension (i.e. ignoring observation and knowledge limitations, discussed later).

Unlike *bona fide* and fiat entities, motivated entities have no official, public extent. For example, a mountain may be accepted publicly as a unique entity, and have an official name, but no official definition of a 'boundary.' Even so, the conventions of language and society usually foster some agreement in the intensions of motivated entities (i.e. people's opinions of the extent of a forest or a historical era may have some differences, but they are not completely different), creating a kind of *common extent*.



However, the unavoidable differences of opinion will make it impossible to define this extent with certainty.

#### 4.1.2 Problems Resulting in Uncertainty

The extent of a fiat or motivated entity can be uncertain whenever its definition/intension) cannot be perfectly reconciled, even conceptually, as shown in Table 1. The definition may be *incomplete*, in which portions of the extent are simply not defined. It may be *incoherent*, in which the referents or the relationships cannot be resolved sensibly as defined. It may be *ambiguous*, in which more than one realization is valid according to the definition and referents. These three problems are often due to a misunderstanding of the reference entities by the agents of creation (those creating fiat entities, or anyone who conceptualizes a motivated entity); they likely did not intend to create a problematic location. However, the observer does not usually have the freedom to 'fix' the definition to what he or she thinks the agents should have done. Incompleteness and ambiguity in motivated entities are frequently the result of the prototyping property of categories (Rosch 1978), in which categories are defined by core examples, not by boundary thresholds; sometimes, there are cases for which

**Table 1** Causes and types of inherent uncertainty in fiat entities

Cause	Spatial Location	Event Time	Attribute Value	Type
Incomplete Definition	mountain defined by peak alone	legislation gives no date of effect	60–70°F 'warm', 80 + °F 'hot' (70–80°F?)	
Incoherent Definition	boundary 'along ridge line' (no ridge line)	'January 32nd'	'appointee' not a real person	Indefinite
Ambiguous Definition	boundary 'along ridge line' (two ridge lines)	15 Jan 1661 (25 Jan 1662?)	60–80°F 'warm', 70 + °F 'hot' (70–80°F?)	
Measure from Indefinite entity	Region bounded by indefinite 'mountains'	end of Ice Age from indefinite 'cold'	total population of indefinite region	
Vague Definition	boundary 'in mountains'	1820 'sort of' part of Industrial Age	hot is more than 'about' 80°F	
Conflicting Definition	A says Ohio in Midwest, B says it is not	two dates of effect in two parts of legislation	Competing leaders during civil war	Vague
Gradual Implementation	part of country gaining some autonomy	date of change in pronunciation of name	pronunciation during change	
Measure from Vague entity	Climate regions from vague 'hot'	Ice Age from vague 'cold'	Population of vague 'Midwest'	

membership cannot be determined. These problems may also occur in rare cases when standard reference systems are unclear, such as the period of change from the Julian to Gregorian calendars in the western world, especially in places like Britain and America where different New Year's Days were in use during the transition.

The type of uncertainty that results from these three causes is an *indefinite* extent. That is, there are points that are neither part of nor not part of the extent set (i.e. their membership is indefinite). In addition, any dependent measurements may also be indefinite. For example, if people are living in an indefinite part of a county, then the total population of the county is indefinite.

The definition may also be *vague*, in which many points may only partially qualify. In motivated entities, this may also be a result of prototyping (Rosch herself debated whether one or both results are valid interpretations of the theory), because some cases are better examples of a category than others, leaving many cases that are only partially members of the category (hence the term 'gray area'). When this vague information function is applied to real points, the extension is also *vague* (Leung 1988; #1 in the list of Robinson and Frank 1985). There may also be *conflicting* definitions: in fiat entities, the definition of one entity may conflict with that of another entity. In motivated entities, conflict happens because the definitions are often personal opinions; thus, each aspect of the entity may contain points that are debatable (Labov 1973). One other problem is when an action is *implemented gradually*. That is, a defined change (say, from A to B) may take place a piece at a time; the implementation interval is somewhat part of the life spans of both A and B.

The type of uncertainty resulting from imprecision, vagueness, and conflict is usually called *gradation* or *fuzziness*, in which the questionable points are only somewhat a part of the entity. Fuzziness was recognized in geographic information by the early 1970s (Gale 1972), although it has only recently begun to gain wide acceptance. Examples of the causes and types of inherent uncertainty in space, time, and theme are summarized in Table 1.

These uncertainties can often be eliminated through arbitration and standardization. The former is often accomplished with administrative jurisdictions such as countries, states, and counties, while the latter is common with themes such as wetlands, soils, land covers, and wildlife habitats. While arbitrated fiat entities have official status (after the date they are arbitrated), arbitrated motivated entities are artificial, losing much of the subtle detail of the real phenomena; one could make the case that they become a fiat entity. In the past, GIS and mapping have relied heavily on arbitrated classifications (wetlands mapping is a common example), but fuzziness-aware models can free us from this reliance.

All of these uncertainties are the result of problems in the information function itself. The control parameters do not seem to be a problem in conceptualization, because the information function is an ideal, without having to take issues into account such as statement precision and discrete sampling.

#### 4.2 *Uncertainty in the Measurement Function*

As defined above, the purpose of the measurement process is to use various techniques to produce a formal model (i.e. the asserted extent) of a conceptual entity. At best, the asserted extent would be a faithful representation of the ideal extent, not of any real-world phenomenon (although if the conceptual entity is *bona fide*, it is identical to a

real-world phenomenon). This 'perfect assertion' would thus include any inherent uncertainties in the conceptual entity.

Since the methods which form the measurement information functions are all imperfect, error and uncertainty (which will be called *assertion uncertainty* to differentiate it from inherent uncertainty) can creep in at any step in the process. Because of this, identification of the cause of uncertainty in the final data can be difficult, especially if one wants to identify the uncertainty present in each phenomenon being modeled.

Fortunately (in this context), sources of historical information are more limited: only those resources that have survived to the present can be used. Barzun and Graff (1985, 166) classify the historical sources into *records*, such as oral, written, and artistic documents intended to record information of historical significance permanently; and *relics*, unintentionally transmitted evidences of past situations, such as archaeological artifacts. For the historical counties of Utah, only the first type of evidence is available and will be considered, although relics also introduce uncertainties that may or may not have a similar nature.

#### 4.2.1 Causes of Assertion Uncertainty Due to Historical Records

When a model is based solely on records, the measurement process can be decomposed into five steps. A record keeper observes real phenomena (1) based on a conceptual entity, and creates a record (2), in which the entity is usually described in reference to other phenomena. Later individuals may preserve or re-record the original record (3), yielding a contemporary record. The current researcher reads and interprets this record (4) and the referenced phenomena to produce geo-historical information, then encodes that information (5) to produce the formal asserted extent. It should be noted that the measurement process for collecting current information, as opposed to historical records, is merely a simplification of this function, with only the first and last steps (although they could be broken down further).

Error and uncertainty may arise whenever there is a problem with one or more of these steps. This was verified by the study examples, in which uncertainty in geographic information gathered from historical records was found to arise for the following reasons (correlations to previous uncertainty theories are referenced):

- *Observation limitations* (1). Any measurement device (including the human senses and mind) has limitations in its accuracy and precision, so information based on its measurements will also be imperfect (Robinson and Frank 1985, 443 #2; Burrough and McDonnell 1998, 222).
- *Lack of evidence* (2, 3). The evidence that has survived and is available to the researcher is always partial and often very unevenly distributed in space, time, and subject matter. This may be due to a lack of preservation, or bias in the choice of phenomena to be observed and recorded. Researchers must extrapolate descriptions of these phenomena from existing evidence, and these estimations will always be less certain than those based on direct evidence (Goodchild 1988, 34; Burrough and McDonnell 1998, 225–226).
- *Lack of reference* (4). When a relative extent description is given, the record-keeper must assume that the reader will be familiar with the extent of the referenced phenomena. However, names of phenomena may change, category semantics may vary, geographic features may disappear, or measurement standards (whether for

dates, locations, or attributes) may be lost. As a result, the researcher may have several possible reference phenomena from which to choose, or may have no idea what the recorder meant.

- *Questionable evidence* (1, 2, 3). Any sources are human artifacts, and are therefore subjective to some degree (Robinson and Frank 1985, 443 #6). While some evidence may be relatively trustworthy, some may be biased, pseudepigraphic, indirect (i.e. an interpretation, translation, or transcription of earlier records), or even mythological. Even so, one may need to use them if better evidence is not available.
- *Conflicting evidence* (1, 2, 3). The primary way to handle questionable evidence is to compare more than one source. However, when this is done, one will frequently be faced with sources that make different assertions, but are equally valid. The conflict is usually due to differing observations of subjective phenomena by multiple record-keepers, or to differing interpretations of ambiguous records by past preservers.
- *Ambiguous evidence* (2, 4). A record can often be interpreted in many ways, usually due to unclear language in the record. Without a better authority, many interpretations may be equally valid.
- *Misinterpretation* (4). The researcher or a preserver can make a mistake in interpreting an extent from a record, whether through lack of skill or rigor, or differences between the recorder and interpreter in their use of language or concepts (Robinson and Frank 1985, 443 #4,7; Gottsegen et al. 1999, 176).
- *Transformation of phenomenon* (3, 4, 5). In any of the interpretive processes, the nature of a phenomenon may be changed, often to make the representation more efficient. Openshaw (1989, 264) lists many transformations that result in error and uncertainty, such as aggregation, classification, and combination. The new conceptual entity may look little like the original conceptual entity, an (albeit necessary) discrepancy.
- *Encoding error* (2, 3, 5). Because the geo-historical information as stored in a computer or in a record is just a representation, it is susceptible to the same errors as any other method of recording information (Robinson and Frank 1985, 443 #3; Burrough and McDonnell 1998, 224). For example, a technician may make mistakes in entering an extent, a computer data model may not model some aspects of the extent (e.g. the presence of vagueness), or a translator may spell a word incorrectly.

#### 4.2.2 Uncertainty in Control Parameters

It would seem that parameters would never be uncertain, since they are chosen at will by the observer. However, any value, even a selected one, is *imprecise*. Even an 'exact' time, such as 8:30:04 GMT 24 January 1954 AD cannot distinguish between times less than a second apart. Temporal parameters are actually small intervals (often termed *chronons*), as are attribute parameters, while spatial parameters are small regions, not points. Thus, the information function is fed a collection of many possible parameter values, resulting in a collection of possible data values. In most cases, the observer would choose a precision fine enough to be inconsequential for the given application, but this is not always possible, especially in historical records.

Another source of uncertainty is the fact that the measurement process is limited in capacity. Most control domains are infinite, but only a finite number of measurements

can be recorded. This necessitates sampling, also called *discretization* (Burrough and McDonnell 1998, 21). These samples are not uncertain themselves, but force the observer to interpolate the data between them, which will be uncertain.

#### 4.2.3 Types of Assertion Uncertainty

The causes listed above can result in three types of uncertainty:

- *Unknown assertion*: the uncertainty is so great that the encoder cannot determine where, when, or how something exists.
- *Imprecise assertion*: the exact datum value is not known, but can be limited to one or more possibilities that hopefully include the correct points. This may be a few distinct possible points (e.g. 'it was either 1 Jan 1850 or 3 Feb 1851') or it may be a region or interval of possible membership (e.g. 'the boundary was somewhere in this area').
- *Inaccurate assertion*: an assertion is in error, even if it appears precise. The final data may hide underlying inherent or assertion uncertainties, or it may just be wrong. This possibility cannot be modeled directly, since the encoder is not aware of it, but may be modeled indirectly using methods such as estimated error fields (Heuvelink 1998). This paper focuses on uncertainties of which the encoder is aware and must deal with, so inaccurate assertions will not be considered further herein.

In past research, some have reused the terms 'vague' and 'indefinite' for the first two cases respectively, but this has only caused confusion between the conceptualization and measurement processes, and those terms are more appropriately applied to the inherent nature of the entities than to our assertions about them. Examples of the above causes in the three aspects, and the types of uncertainty that may result are given in Table 2.

## 5 Forms of Uncertainty

The previous section shows some parallels between inherent and assertion uncertainty, which should aid in representing them formally in the UTEM. However, it is vital that the two be discernable in the resultant information, and that the various types of uncertainty in each process be discernable. Inherent uncertainty will require different interpretations and different analysis than assertion uncertainty, so the information user must know which is present.

Many approaches could be used to describe the forms that the two kinds of uncertainty can take. The UTEM proposes a general extension of Fuzzy Set Theory (Zadeh 1965). It has proven to be of value in geographic applications (even proposed for spatiotemporal information by Pfoser and Tryfona 2000, but not implemented), although it does have shortcomings, most notably its inability to distinguish between different types of uncertainty, and the difficulty in implementing it (i.e. finding exact membership values). The UTEM extension overcomes the first shortcoming; the second is more difficult, but also manageable. Fuzzy Set Theory is based on looking at the possible answers to a basic proposition of set theory, which can be applied to any set  $X$  (in this case, the domains of space, time, and each attribute):

**Table 2** Causes and types of assertion uncertainty

Cause	Spatial Location	Time/Date	Attribute Value	Type
Measurement Limitation	Lat/Lon from stars	Birth date from later memory	Temperature in whole degrees	imprecise
Lack of Evidence	Portion of boundary not described	No record of birthdate	Decennial Census (what was 1865 pop?)	imprecise unknown
Lack of Reference	'North of X River,' (where is X River?)	'The third year of the reign of X' (when X?)	'The son of X' (who was X?)	imprecise unknown
Questionable Evidence	'The great king ruled the lands of the five rivers'	'born when the moon was in Cancer'	'His army of 100,000 conquered all'	imprecise unknown
Ambiguous Evidence	'Along a large river' (which river?)	'The day of the great feast' (which feast?)	'The son of X' (which X?)	imprecise
Conflicting Evidence	A: 'to the X River' B: 'to the Y River'	A: 'in 1850' B: 'in 1851'	A: 'army of 20,000' B: 'army of 30,000'	imprecise
Misinterpretation	'All of Asia' (recorder: exc. Arabia, encoder: inc. Arabia)	'In the Industrial Age' (recorder: IA > 1820, encoder: IA > 1770)	'The income was high' (recorder: high > \$300, encoder: high > \$8000)	inaccurate
Transformation	People → Census tracts	Years → 'Information Age'	Income values → 'high,' 'low'	imprecise inaccurate
Encoding Error	Digitizing error	Entered wrong year	Typo in name	inaccurate

$$P: p \in X \tag{2}$$

In the context of an entity extent, this could be rephrased as the question, 'is the point  $p$  part of the datum  $d$  [which as stated earlier could be a single point or a large set] or not?' Since the proposition has a truth value for every  $p$ , the set as a whole can be described as a *membership function*:

$$m_X: X \rightarrow M$$

$$m_X(p) = \text{truth of } P \tag{3}$$

In classic set theory, the truth domain  $M$  has only two values, true and false. Fuzzy Set Theory allows for a more broad domain, and the UTEM extends this concept further. Now the ideal datum of Equation 1 can be defined as a *qualified datum*  $d^*$ :

$$d^* = \{ \langle p, m_X(p) \rangle \} \tag{4}$$

### 5.1 Form of Ideal Extent

As discussed above, inherent uncertainty can be one of two types, *indefiniteness* and *vagueness*. These result in two types of truth values for the proposition in Equation 2:

- *Vague Extent*  $\rightarrow$  *Partial Membership* (P is somewhere between true and false,  $m_X(p) \in (0,1)$ ). In addition to being completely in or out of the set, it is possible for a point to only partially be a part of the phenomenon's extent. This is the domain employed in Fuzzy Set Theory.
- *Indefinite Extent Null Membership* (P has no valid truth value,  $m_X(p) = \omega$ ). In this case, a point is neither in nor out of the extent to any definable degree;  $\omega$  is an arbitrary symbol sometimes used to represent this null state. Null values have been used in databases to model uncertainty as a whole (Codd 1979), and the three-valued logic necessary to reason with null values is well developed. Methods for manifesting three-valued logic in set theory, such as Rough Sets (Pawlak 1982) are also available, but the applicability of these methods to the semantics of indefiniteness needs to be investigated further.

Thus, the total range of M is the union of the certain domain  $\{0,1\}$  with these two domains:

$$M = [0, 1] \cup \omega \tag{5}$$

### 5.2 Form of Asserted Extent

As with the ideal extent, the asserted extent set of a phenomenon can be modeled to handle uncertainty and indeterminacy by looking at the proposition of Equation 2. However, the assertion is not made of the extent of a real phenomenon, but of a conceptual entity, for which each aspect is manifest by a qualified datum  $d^*$ . Therefore, the assertion proposition needs to be phrased with respect to the membership function rather than crisp membership, as follows:

$$P^*: m = m_X(p) \tag{6}$$

Associated with any combination of point and membership value is a corresponding *validity*, which represents the degree to which the observer knows that this proposition is true. Applying this principle to all possible combinations of point and membership yields a *validity function*  $v$ , which is analogous to the membership function:

$$v_X: X \times M \rightarrow V$$

$$v_X(p, m) = \text{truth of } P^* \tag{7}$$

where V is the range of possible validity. As with the membership function and inherent uncertainty, the different types of uncertainty correspond to different possible values of  $v$  that comprise V. Worboys (1998, 97) uses a proposition similar to Equation 2 to handle imprecise assertions, with three possible values in V: 'yes,' 'maybe,' and 'no.' In this work, more detail is necessary:

- *Known Assertion*  $\rightarrow$  *Exact Validity*.  $m_X(p) = m \Rightarrow v_X(p, m) = 1$ ,  $m_X(p) \neq m \Rightarrow v_X(p, m) = 0$ . If the encoder is sure of the extent of an entity, then each possible

membership at any given point is either true or false. Of course, just because the encoder is ‘sure’ does not imply that the encoder cannot be wrong (i.e. an inaccurate assertion as defined above).

- *Imprecise Assertion → Partial Validity.*  $v_X(p, m) \in (0,1)$ . If the encoder believes that a membership value may be correct, then an intermediate value can be assigned, corresponding to one of many available measures of the degree to which it is known that  $m$  is the correct membership. It may be a probability (if the range of possible memberships is continuous, it could be a *confidence function*, measuring the probability over a small interval around each membership value, rather than a true probability density function). Other interpretations are possible, including possibility (Dubois and Prade 1988) and the belief function of Evidence Theory (Shafer 1976), which may be very useful in the specific case of historical information because it allows for updating belief based on new evidence. According to Smets (1994), none of these options is globally ideal; each is useful in specific situations. In any case, the validity function as a whole conforms to none of these theories (because the membership of each point is an independent event), but is a fuzzy set and conforms to Fuzzy Set Theory.
- *Unknown Assertion → Null Validity.*  $v_{Eg^*}(\bar{p}, m) = \omega$ . The encoder cannot determine whether the membership is valid or not. This option can be removed by representing unknown assertions by an even partial validity function across the rest of  $V$  (Dey and Sarkar 1996), but is left in for symmetry.

Thus,  $V$  is identical to  $M$ :

$$V = [0, 1] \cup \omega \tag{8}$$

The result of the measurement information function is thus an *asserted datum*  $d^{**}$ :

$$d^{**} = \{ \langle p, m, v_X(p, m) \rangle \} \tag{9}$$

Table 3 shows the possible combinations of values for  $m$  and  $v$ , and the proper interpretation.

In the validity function, fuzzy set theory is being used to model something other than fuzziness. This is valid, because the theory is not a philosophical explanation of vagueness, merely a formal method for handling degrees of membership in a set. Methods such as Possibility Theory use the same formalisms, but with different semantics. However, it is vital that one be careful in interpreting the formal constructs; consequently, a sound data model would need to encode the choice of measure for each validity function.

The asserted extent in each aspect is similar to a *type-II fuzzy set* (Klir and Yuan 1995, 17), with the addition of the null values to the function domain. This addition may seem simple, but it adds a great deal of complexity to the use of the set. A theory of null-membership sets has not been developed to the same degree as classic and fuzzy sets, although three-valued logic and rough sets (Pawlak 1982), which handle an intermediate level of membership (which is different than, but may work the same as, null membership) may prove useful.

A complete logic and algebra for combining fuzzy and null-membership sets does not currently exist. More research is needed to develop such a model to handle all three aspects of the geographic universe and to handle both the membership and validity functions simultaneously.



**Table 3** Types of uncertainty in the Uncertain Extent Set Model.  $p$  can be a point in either the temporal, spatiotemporal, or temporal attribute domains of Equation 2. In the validity function,  $m$  represents all possible membership values in  $M$  but those specified.

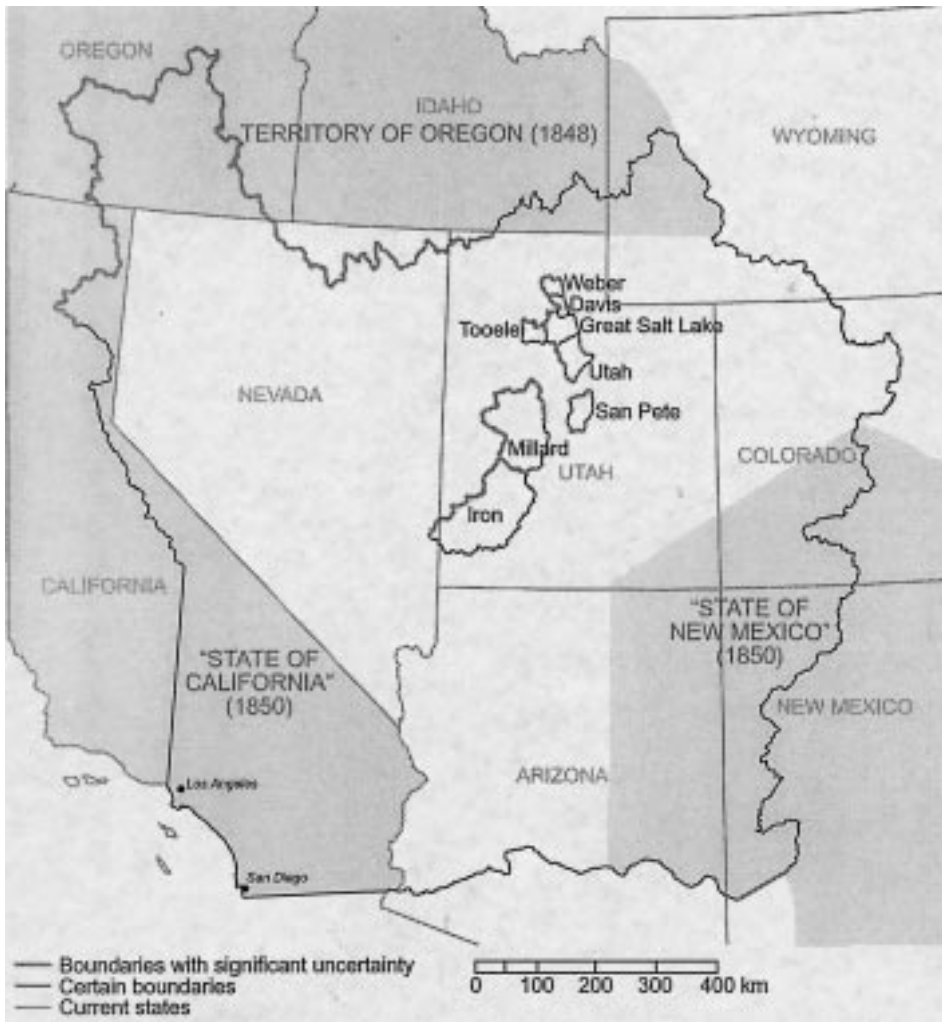
Inherent Uncertainty	Assertion Uncertainty	Validity Function	Interpretation
Exact	Known	$v(p, 1) = 1; v(p, m) = 0$	'I know this point is in $g'$
Vague	Known	$v(p, 0.5) = 1; v(p, m) = 0$	'I know this point is 50% part of $g'$
Indefinite	Known	$v(p, \omega) = 1; v(p, m) = 0$	'I know it is not possible to determine whether this point is part of $g$ or not'
Exact	Imprecise	$v(p, 1) = 0.7; v(p, 0) = 0.3; v(p, m) = 0$	'This point is probably in $g$ , but it may be out'
Vague	Imprecise	$v(p, 0.5) = 0.7; v(p, 0.3) = 0.3; v(p, m) = 0$	'This point is probably 50% part of $g$ , but it may be 30% part of $g'$
Indefinite	Imprecise	$v(p, \omega) = 0.7; v(p, 0) = 0.3; v(p, m) = 0$	'This point is probably impossible to determine, but it may be completely out of $g'$
Exact	Unknown	$v(p, 1) = \omega; v(p, 0) = \omega; v(p, m) = 0$	'This point is either in or out of $g$ , but I don't know which'
Vague	Unknown	$v(p, \omega) = \omega; v(p, m) = 0$	'I don't know how much this point is part of $g'$
Indefinite	Unknown	$v(p, \omega) = \omega; v(p, m) = \omega$	'I have no idea of the status of with this point'

## 6 Application of the Uncertain Extent Set Model

The above taxonomy can be used to evaluate the uncertainty present in any assertion of lifespan, location, or property. As an initial test of its applicability, the historical Utah county boundaries were completely classified. Although administrative units like counties are generally precisely defined and accurately surveyed, a variety of uncertainties were discovered in the Utah study. This was especially true in the first few years of white settlement, during which the region was governed by the provisional (but functional) State of Deseret (1849–1851) and the Territory of Utah (1851–1895). Both governments created and modified counties, many of which were fraught with uncertainty in their locations, lifespans, and properties (Figure 1). Three particular cases are described below.

### 6.1 East Boundary of Weber County, 1850–1852

Weber County was created in January 1850, with an extent defined as 'all that portion of country known as Weber Valley, and extending as far south as Stony Creek, and



**Figure 1** The provisional State of Deseret (1849–51), with counties created by the provisional government, showing boundaries with significant positional uncertainty (more than 500 meters, several kilometers for many) and overlapping territorial claims

west to the Great Salt Lake’ (Morgan 1940, 180). The eastern and northern boundaries are implied by the term *valley*, which is also used to delineate the other counties created in this act. In common usage, this term can have many meanings; this potential for confusion was recognized by the legislators, who included a definitive clause in the act:

‘Sec. 17. Whenever a County is mentioned as including a valley, the boundaries of the same shall extend to the natural boundaries of said valley, the summit of the surrounding mountains, on the highest dividing ridge between said valleys’ (Morgan 1940, 181).

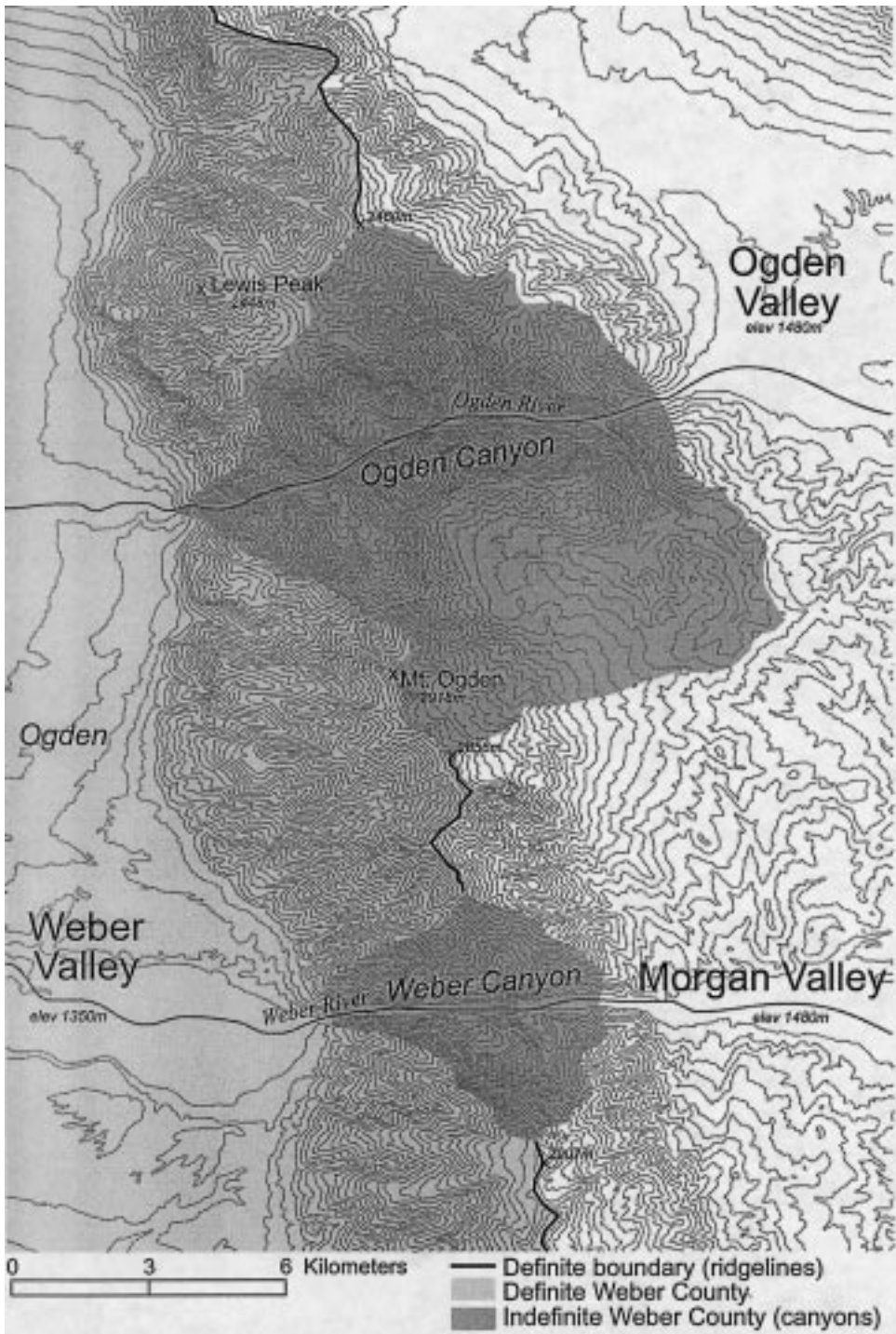
In most cases, this definition is not problematic, and ridges normally make good boundaries. However, sometimes the definition is difficult to apply. For example, the mountain ridge east of Weber Valley is cut by two canyons containing rivers coming from other valleys, as shown in Figure 2. The canyon has no 'dividing ridge,' and thus the definition cannot be applied. In 1952, the boundaries of the county were completely redefined to more easily (i.e. better) defined locations. This situation can be categorized by the framework above as follows:

- *Geo-historical Aspect in Doubt*: location:  $l = f(e, t)$ ;  $e = \text{Weber}$ ,  $t \in [1-17-1850, 3-1-1852]$
- *Problem in Conceptualization Function*: incomplete definition. The definition does not explain what to do in the canyons.
- *Form of Ideal Extent*: Indefinite extent/Null membership: The canyons themselves are neither in nor out of the county.
- *Problem in Measurement Function*: Negligible, but possibly lack of evidence. The complete law creating the county is available, but there is a remote possibility that the residents or lawmakers intended the boundary to follow particular ridges within the canyons, which information is not currently available.
- *Form of Asserted Extent*: Exact Validity. We know for sure that the canyons are indefinite.
- *Validity Function*: For all spatial points  $p$  in the canyons  $v(p, \omega) = 1$ ;  $v(p, m) = 1$  for all other  $m \in M$ .

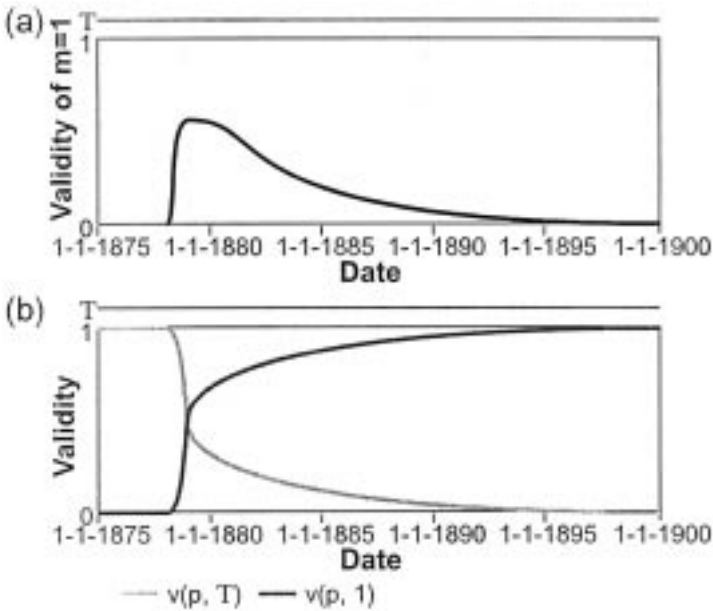
## 6.2 Survey of Morgan/Summit Boundary

Morgan County was created on January 17, 1862 (Utah Territorial Legislative Assembly 1862, 50) bounded on the south by Summit County; the latter's northern boundary was defined in the same act as, 'the summit of the range of mountains forming the upper kanyon [sic] of East Kanyon [sic] Creek.' This canyon-based boundary created an indefinite extent in much the same way as the previous example. The canyon was an important transportation route and source of water for both counties, so the indefinite boundary caused a lengthy dispute. By 1872, the counties had agreed to settle the issue by hiring Jesse Fox, the territorial surveyor, to arbitrate. However, surveyors had no legal right to settle an unclear county boundary until the legislature passed an authorizing law in 1878 (Utah Territorial Legislative Assembly 1878, 20), that was apparently sponsored specifically to resolve this disputed boundary. No record of a survey has survived in either county, nor any record of either county court accepting such a survey. The text of the law has never been clarified. However, it is clear on maps of the early 1900s that the boundary had been clarified. Thus, it is presumable that a survey was performed sometime between 1878 and 1900, most likely as soon as possible after the law was passed (in mid-winter).

- *Geo-historical Aspect in Doubt*: time. In this case, the temporal parameter sample that marks the end of the indefinite location and beginning of a definite location is an event (the survey being accepted by the county and state governments) rather than a point in time. Thus, the time is a measurement:  $t = f(v)$ ;  $v = \text{survey acceptance}$
- *Problem in Conceptualization Function*: none. The definite boundary became effective the moment the map was signed by the authorities.



**Figure 2** Indefinite portions of 1850 Weber County boundary, due to the inapplicability of the definitive term, 'highest dividing ridge.' Contour interval is 50 meters



**Figure 3** Hypothetical graph of the uncertain time of the survey of the Morgan/Summit county boundary: (a) modeled as an uncertain time measure of a change event; and (b) modeled as a changing measure of the location of Morgan County for any point in the canyon that was eventually on the north side of the survey line

- *Form of Ideal Extent:* exact/crisp membership. Any times were either after the survey was accepted or before.
- *Problem in Measurement Function:* lack of evidence. By law, a survey map had to be produced and signed, but such a map appears to have been lost.
- *Form of Asserted Extent.* Imprecise assertion/partial validity. Times during the 1878–1900 period have some probability of being the date of the survey.
- *Validity Function.* The event would be represented as a confidence function, showing probability rapidly increasing from the winter to spring 1878, and gradually declining from 1878 to 1900, as shown in Figure 3a. An alternative approach would be to forego the event abstraction, and use a single validity function for the location, in which points in the canyon have gradually decreasing validity for  $\omega$ , and increasing validity for either 0 or 1, as time progresses past 1878, as shown in Figure 3b.

### 6.3 The Governed Population of Deseret, 15 June 1850

Because the State of Deseret was created unilaterally by the Mormon community, its boundaries and government were not universally accepted by other governments, or even many people living within its claimed boundaries, as shown in Figure 1. Oregon Territory (which overlapped the northern periphery) had been operating since 1848, and California and New Mexico had both set up their own provisional governments with overlapping boundaries. During this time, the Mormon population that paid

allegiance to Deseret was concentrated along the base of the Wasatch Mountains (roughly the area divided into counties in Figure 1). The remainder of the 'state' was sparsely populated by Native Americans who paid little attention to white governments; the only other sizable White population in the claimed area were the Mexicans at San Diego, most of whom had little interest in becoming part of California nor Deseret; there was a small community of Mormons remaining in San Diego from the Mexican War, interested in establishing the town as a port with access to Salt Lake. This ethereal nature of the state makes the measurement of the 'effective' population a complex proposition. Due to the lack of U.S. enumerators, a census was taken by the Church in 1850, obviously only counting church members in the core region.

- *Geo-historical Aspect in Doubt*: 'governed' population; that is, the population of the 'region effectively controlled by the Deseret government' (a motivated entity):  $a = f(e, t)$ ;  $e$  = Deseret-controlled zone,  $t = 6/15/1850$ . This is thus a dependent measure.
- *Problem in Conceptualization Function*: measure from vague entity. The zone of control is vague, due to conflicting definition (conflicting between provisional governments, and between Deseret and its 'residents'). For the region, the area around Salt Lake City (zone A) would have a membership of one, the otherwise-unclaimed Indian-occupied territory (B) a fairly low value, and American- and Mexican-settled areas in the claimed areas of New Mexico and California (C) a membership of practically zero. Residents of each area would be assigned matching 'degree of governance' attributes, and then counted.
- *Form of Ideal Extent*: Vague. There would be at least three (depending on the degree of detail) candidate populations. The population of zone A would have a membership of nearly one, that of A + B a small membership, and that of the entire state almost zero.
- *Problem in Measurement Function*: lack of evidence. A census of one small region would be several months old by June, and would require extrapolation. Indian, Mexican, and American populations elsewhere would be rough estimates based on very general evidence.
- *Form of Asserted Extent*. Imprecise assertion/partial validity. Each of the three candidate populations from the ideal extent would only be determinable to a sizable interval (which would be most refined for zone A), which might even overlap one another. One might be able to generate a normal probability distribution for the estimates of each of the three populations.
- *Validity Function*. Quite complex. A large group of possible populations would have varying degrees of validity for each of the three memberships. For example:

$$v(36000, 0.9) = 0.6$$

$$v(36000, 0.07) = 0.8$$

$$v(36000, 0.03) = 0.3$$

This final example illustrates the most commonly stated shortcoming of Fuzzy Set Theory: the exact membership values (0.9, 0.07, 0.03) are completely arbitrary, not being an actual measure of anything. In most applications, this is true, but fuzzy sets can still be used (somewhat more weakly) if membership is interpreted as an index (essentially a very detailed ordinal scale) rather than a quantitative measure.

## 7 Conclusions

The Uncertain Temporal Entity Model describes, in a formal way, the variety of causes, types, and forms of uncertainty present in geo-historical information (and likely, with some extension, other kinds of geographic information as well). The model has two important implications.

First, it gives a clearer picture of the nature of geo-historical information. This allows the person compiling this kind of information to identify exactly what is happening in any situation in which uncertainty arises: what is causing the uncertainty, where it lies, and what it looks like. This should result in more rigorous research practices and more accurate reporting of data and results. The users of geo-historical information will also benefit from this clearer understanding. In the Utah example, reporting the uncertainty inherent in historical county boundaries can aid historians and genealogists (the primary users of the information) in their research. For example, reporting the uncertainty in the jurisdiction of an area between two counties would alert a researcher to look in both counties for civil records.

Second, the framework suggests a strategy, or at least guidelines, for modeling uncertainty and indeterminacy in digital stores (i.e. GIS). Just as the compiled data should be stored in a permanent database with as little abstraction and preprocessing as possible (Goodchild 1988, 42), so should the uncertain aspects of those data be stored as faithfully as possible. In addition, storing the semantics of each uncertainty in the database (i.e. not just the amount of uncertainty in an entity, but what is going on and why) will enable more accurate analysis.

This framework does not yet qualify as a complete theory of the nature of uncertainty. Such a theory will require several more elements:

- Testing of the applicability of the framework in more situations, including other temporal domains such as current dynamic phenomena (e.g. weather), future developments (e.g. transportation planning), and prehistoric phenomena (e.g. geologic processes). These use sources other than historical records, such as archaeological relics, geological studies, and present-day data collection techniques. When compared to uncertainty literature based on other domains, the UTEM appears to cover everything, but it may not. For example, error fields are commonly used for describing uncertainty in continuous fields such as temperature (e.g. Heuvelink 1998), but they may not be handled very well by the UTEM.
- Further investigation into the role of relationships and other constraints in assertions of extent. In some cases, this relative knowledge may outweigh any knowledge of absolute position (e.g. we know A happened before B, but we do not know when they occurred). In the least, they will probably enable more efficient storage (e.g. two adjacent regions share the same uncertain zone, so its location only needs to be entered once). In addition, these relationships may themselves be uncertain, in ways not currently considered by the UTEM.
- Development of a GIS data model based on the Uncertain Temporal Entity Model. One common complaint about fuzzy sets is that they are difficult to implement in discrete systems such as GIS. However, the UTEM frequently shows large-scale patterns in uncertainty that could lead to ‘shortcuts’ in implementation. An implementation is currently under development that has been successful so far and will be reported at a later date.

The current framework may very well suffice as a theory, but additional categories of uncertainty may be discovered. Either way, the framework presented herein provides a solid foundation for understanding and managing uncertainty in spatiotemporal information.

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