Representation and Spatial Analysis in Geographic Information Systems

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A common—perhaps modal—representation of geography in spatial analysis and geographic information systems is native (unexamined) objects interacting based on simple distance and connectivity relationships within an empty Euclidean space. This is only one possibility among a large set of geographic representations that can support quantitative analysis. Through the vehicle of GIS, many researchers are adopting this representation without realizing its assumptions or its alternatives. Rather than locking researchers into a single representation, GIS could serve as a toolkit for estimating and exploring alternative geographic representations and their analytical possibilities. The article reviews geographic representations, their associated analytical possibilities and relevant computational tools in the combined spatial analysis and GIScience literatures. The discussion identifies several research and development frontiers, including analytical gaps in current GIS software. *Key Words: geographic information systems, geographic representation, spatial analysis.*

A common—perhaps modal—representation of geography in spatial analysis (SA) and geographic information systems (GIS) is native (unexamined) objects interacting based on simple distance and connectivity relationships within an empty Euclidean plane. This representation has essentially remained untouched since its inception during the birth of quantitative geography and computer cartography, an era characterized by scarce geographic data, weak computers, and crude spatial algorithms. It is only one possibility among a large set that can support "analysis," or the ability to measure and infer quantitative properties and relationships.[1] The Euclidean model is useful in many contexts, such as cartography, navigation, and many types of analysis. However, there may be latent explanatory power in geographic space not captured by the Euclidean model and consequently missed by SA techniques and GIS-based analysis based on this georepresentation. Representation and analysis are closely linked: mathematical and computational tools, however powerful, cannot extract more information than is latent in a representation.

Through the vehicle of GIS, many researchers are adopting the Euclidean model and its related analytical possibilities without realizing its assumptions or its alternatives. Rather than locking researchers into a single representation, GIS could serve as a toolkit for estimating and exploring alternative geographic representations and their analytical possibilities for a given geographic phenomenon or problem. Reconsidering and expanding the geographic representation model underpinning both SA and GIS is an unexplored avenue for improving analytical capabilities of both.

Theories and techniques for alternative geographic representations and analyses exist in the SA and geographic information science (GIScience) literatures.[2] SA has a small but steady current of literature on representation issues in analysis, including theories of geographic space and techniques for estimating distance metrics, analyzing geographic relationships between spatial objects, and analyzing geometric form. The GIScience literature contains recent breakthroughs in computational methods for storing and processing digital representations of geography, including progress on several representational issues of long-standing concern in SA. These literatures are mostly independent from each other, judging by the low level of cross-citations. The potential synergy of the complementary developments in both literatures may not be apparent to researchers in either subfield or to researchers in broader domains of human and physical geography as well as related disciplines. The fusing of these independent research trajectories into an integrated research agenda could generate SA and GIS tools that might help reveal new insights into the role of geography in explaining many physical and human phenomena.

This article reviews geographic representations, their associated analytical possibilities, and relevant computational tools in the combined SA and GIScience literatures. We discuss developments in the SA and GIScience literatures from the perspective of the Beguin-Thisse

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theory of geographic space (Beguin and Thisse 1979). The Beguin-Thisse theory makes explicit the fundamental assumptions and properties required to measure geographic phenomena in a manner that can support analysis. It also clearly illustrates other analytical georepresentations and the requirements for constructing these representations. We also consider complementary conceptualizations of fundamental geographic relationships and form from the SA and GIScience literatures. In addition, the discussion highlights analytical gaps in current GIS software, suggesting requirements for GIS software development.

We begin the review by describing in detail the motivation for this article: tracing issues regarding representation and analysis in GIS and SA (second section). In the third section, we discuss the measurement and analysis of fundamental geospatial properties, in particular location, length, and area. We show that the Euclidean model is only one among many possible frameworks for representing these properties. In the fourth section, we discuss the measurement and analysis of geographic attributes, the geographic relationships and properties implied by these attributes, and the possibilities for analyzing these attributes. The fifth section provides concluding remarks regarding research and development frontiers, as well as strategies for progress along these frontiers.

Representation Issues in Spatial Analysis and GIS

Spatial Analysis and GIS

Spatial analysis (SA) is a subfield of geography and regional science that studies properties that vary with geographic location (see Goodchild 1987). The primary methodological approach is quantitative analysis (see Taaffe 1974). Geographic location is also a central organizing principle in geographic information systems (GIS) and science (GIScience). A shared goal of SA and GIS is to improve capabilities for understanding geographic phenomena and solving geographic problems.

Given their mutual focus on geographic location as an organizing principle, there have been many calls and attempts to build stronger linkages between SA and GIS (e.g., Goodchild 1989, 1992; Fotheringham 1991; Fischer and Nijkamp 1992; Fotheringham 1992, 1993; Goodchild et al. 1992; Fotheringham and Rogerson 1994; Turner, Meyer, and Skole 1994; Openshaw 1994, 1995). While much progress has been made, criticism persists that linkages between SA and GIS are not fully realized (Anselin and Getis 1992; Batty 1992; Grossmann and Eberhardt 1992; Anselin, Dodson, and Hudak 1993; Goodchild, Parks, and Steyaert 1993; Bailey and Gatrell 1995; Miller 1999; Fotheringham, Brunsdon, and Charlton 2000; Ungerer and Goodchild 2002). Although some techniques, such as local spatial statistics and geographically weighted regression, are difficult, if not impossible without GIS (Anselin 1995; Brunsdon, Fotheringham and Charlton 1996; Getis and Ord 1996), the spatial-analytic breakthroughs seem sparse relative to the apparent possibilities.

A similar, sometimes implicit criticism is apparent in domain-oriented subfields of human and physical geography. In many subfields of physical geography—primarily biogeography and landscape ecology—GIS is embraced as a tool for storage, display, data manipulation, and integration. The use of GIS as an analytical toolkit is less common (see, for example, Walsh, Butler, and Malanson 1998; Noonan 1999; Aspinall and Pearson 2000; Newson and Newson 2000; Mawdsley 2001). Climatologists and meteorologists have largely ignored GIS due to its lack of capabilities for analyzing temporal processes, a limitation frequently described in the literature (Langran 1993; Peuquet 1994, 2001; Dragicevic and Marceau 2000). Some human geographers have commented explicitly that "Cartesian perspectivalism" has limited the potential of GIS (Roberts and Schein 1995; Curry 1998). A sterile geometry is associated with a simplified GIS that fails to fully represent some segments of society or complex geographic processes.

Representation in Spatial Analysis and GIS

SA and GIS have roots in the "quantitative revolution" in geography and computer-assisted cartography, respectively. Both emerged in an era when data were relatively scarce and computational power was relatively limited (1950s–1960s). Researchers in both areas adopted a similar geographic representational model; we refer to this as the *Euclidean model*, since it highlights simple Euclidean space properties. The Euclidean model represents geographic entities as points, lines, and polygon objects, or as an intensity field, within an empty Euclidean planar space. These objects are often treated as native, in the sense that their form is accepted rather than examined (with the exception of the well-known modifiable-areal-unit problem; see Openshaw and Taylor 1979). Although SA and GIS evolved somewhat separately, planar Euclidean geometry remains central to the representational model in both fields.

In SA, the Euclidean model allows tractable analytical calculations based on simple objects, Euclidean distance functions, and basic connectivity relationships that operate within a conveniently empty space. These properties are easily extracted from maps. In GIS, the Euclidean model is convenient for data display, navigation, and other traditional cartographic applications. It also does not create problems for maintaining large databases in inventory and management applications, another traditional use of GIS.

Since the formative years of SA and GIS, data have become less scarce and computational power has dramatically improved. Of course, computers will always have fundamental limitations with respect to power (e.g., intractability; see Garey and Johnson 1979) and scope (e.g., properties that are difficult to measure, such as "political power," or data that are controlled for security, profit, or privacy). Nevertheless, given the new data-rich and computation-rich environment for research and practice, it is worthwhile to reexamine the fundamental activities in the research and application communities. Can we use the new data and computational power to do things differently, perhaps even better? This is not an original message; other attempts to rethink current practice in light of new data and power include computational human geography (Openshaw 1994), geocomputation (Longley 1998; Openshaw 2000), geographic (and other) data mining (Miller and Han 2001), agent-based computational economics (Tesfatsion 2003), and bioinformatics (Baxevanis and Ouellette 2001), just to name a few.

Representation and Analytical Possibilities

There is an inherent tension between representation and analysis. Analysis is based on a selective approximation of reality: we purposely ignore things we think are peripheral to the phenomenon being modeled. The Euclidean model may be sufficient as a selective view of geographic reality: distance and simple connections between native objects in empty Euclidean space may capture sufficiently the explanatory power of geographic space for many human and physical geographic processes. However, these assumptions have not been examined in a comprehensive and systematic manner in many subfields of geography and related disciplines.

A major and broad-ranging research question raised by this article is that of whether there are particular geographic phenomena or problems for which alternative representations and analysis can lead to new insights. Possibilities include phenomena that are highly dependent on spatial interaction among disaggregate and geographically dispersed entities. For example, Cliff and Haggett (1998) argue that the Euclidean plane is limited for analyzing spatial diffusion processes such as the spread of disease over geographic space. Geographic representation can limit and even bias the analysis of spatial interaction systems (Worboys, Mason, and Lingham 1998; Horner and O'Kelly 2002). Geographic representation can lead to qualitatively different outcomes from those produced by interspecies population dynamics models in ecological analysis (Durrett and Levin 1994; Malanson and Armstrong 1997; Malanson 2002).

Closely related possibilities are phenomena the evolution of which is sensitive to initial conditions and therefore measurement of geographic context. Physical processes such as weather systems and climate change are well-known examples of chaotic dynamic processes, in which small differences in starting conditions lead to large differences in solution trajectories; this is often referred to as *deterministic complexity* (see Manson 2001). Human systems can also exhibit sensitivity to geographic context. For example, economic phenomena that are subject to increasing returns (or "positive feedback") can exhibit *path-dependency*, or processes with many possibilities, rather than a stable and predicable outcome, as with traditional equilibria (Arthur 1994). Fujita, Krugman, and Venables (1999) use increasing returns within a monopolistic competitive framework to solidify the economic foundations of location theory and reconcile von Thünen's land use theory with Lösch's central place theory. They conduct a simple experiment to analyze the impact of geographic variation on location patterns. Although their analysis is highly abstract with respect to geographic representation (a simple tree network), it clearly demonstrates the strong influence of geographic context on the evolution of the spatial economy. In a review of complexity theory and its geographic applications, Manson (2001) notes that social theorists in human geography have implicitly embraced deterministic complexity and the importance of local context and interaction in studying phenomena such as economies, social organization, and cities.

Another potential opportunity for alternative geographic representation and analysis is enhancing disaggregate spatial statistics such as G and G* autocorellation statistics (Ord and Getis 1995) and geographically weighted regression (Brunsdon, Fotheringham, and Charlton 1996). Disaggregate spatial statistics usually require some form of a distance weight matrix; this is typically calculated from the Euclidean plane and therefore carries with it the assumptions and limitations of the Euclidean model. Another problem is that matrices become bulky when large datasets are being evaluated. Although kernels are frequently applied to narrow the focus to a smaller geographic region, the calculations can still be cumbersome. Brute-force technology certainly can handle these calculations, but a non-Euclidean geographic representation may provide elegant and unexpected alternatives.

Yet another domain in which alternative geographic representations may generate new breakthroughs is *continuous spatial modeling*. Continuous spatial modeling views two-dimensional geographic processes as an intensity field rather than a collection of discrete locations and spatial objects. Geographic phenomena such as markets, and transportation and urban systems are rooted in an underlying fabric of least-cost path relationships among continuous locations in space (Puu and Beckmann 1999). Although continuous spatial modeling has its roots deep in the SA and regional-science tradition (e.g., Warntz 1965; Werner 1968; Wardrop 1969; Angel and Hyman 1976), it has fallen out of favor in more recent times. This is not due to a perceived intellectual failure of continuous spatial modeling. Rather, it is more likely due to the rise of the digital computer, its affinity for discrete structures, and the subsequent biasing of the human sciences toward objects rather than fields. Although object-based representations seem more natural for many human phenomena, the field approach can provide unique and complementary insights (Puu and Beckmann 1999). The digital computer could breathe new life into continuous spatial modeling by supporting estimation and analysis of sophisticated least-cost path and other geographic relationships in both empty and attributed geographic space, as well as supporting integrated object-field based representation and analysis (see Cova and Goodchild 2002).

A Framework for Geographic Representation and Spatial Analysis

The central framework of this article is a conceptual model of geographic representation based on the Beguin-Thisse theory of geographic space (Beguin and Thisse 1979), described in detail in the third section. Note that there are no widely or explicitly accepted standards for geographic representation in SA or GIScience. We could define a mathematical space through any set of objects and some defined relation between pairings of those objects (see Gatrell 1983). Alternative models of space with special reference to geographic representation are available (e.g., Herring 1991; Schneider 1997; Smith and Varzi 1997; Casati, Smith, and Varzi 1998). However, the Beguin-Thisse theory focuses on the properties required to support the quantitative measurement and analysis of attributes using space as a framework. Since SA requires quantitative measurement of geographic properties (by definition and in practice), the Beguin-Thisse system is a reasonable statement of the (often unstated) representation assumptions made by spatial analysts.

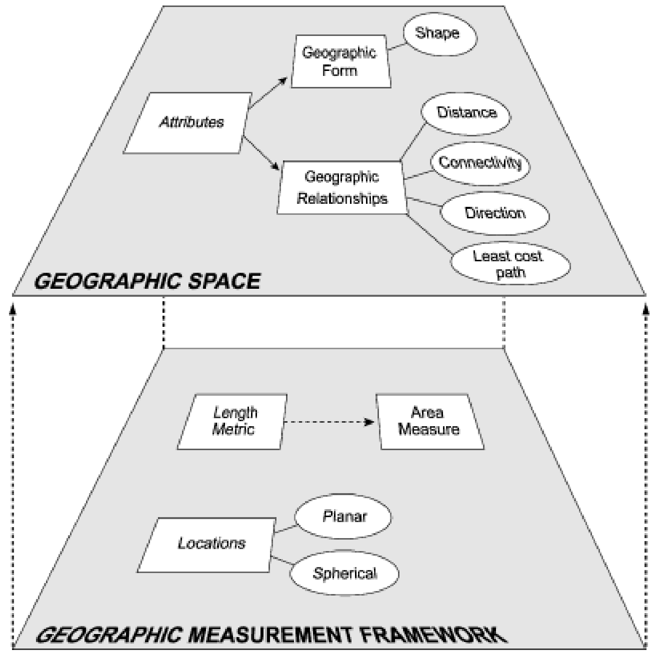

Figure 1. A conceptual model of geographic space.

Figure 1 illustrates the major components of the representational framework. The foundation is a *geospatial measurement framework*, defined by a set of locations and a length-metric for spatial measurement. The geospatial measurement framework supports *geographic space*, created by measuring attributes within the metric implied by the framework. From geographic space emerges the properties *geographic relationships* and *geographic form*. For each component illustrated in the diagram, we describe formal representations, the analytical functions they support, and tools for implementing these functions in a computational environment such as a GIS. We also comment on the shortcomings of widely available GIS software.[3]

Geospatial Measurement Framework

The Beguin-Thisse theory dictates that measurement of geographic attributes requires an underlying geospatial measurement framework (GMF; termed *pre-geographic space* by Begin and Thisse [1979]). We prefer the term "geospatial," as opposed to "spatial," to highlight that this framework ultimately—if only conceptually—relates to locations and relations on the Earth's surface. A GMF is defined through a set of locations, a length-metric relation between locations and an area measure:

$$(X, d_L, \mu_A) \quad (1)$$

where X is a set of locations with at least two locations distinguishable from each other, d_L is a length-metric defined over length dimension $[L]$ drawn from a set of nonnegative real numbers, and μ_A is an area measure defined over the Cartesian product of the length measure $[A] \equiv [L]\times[L]$.

Geospatial Measurement Framework Components

Locations. The set of locations can be either *strictly bounded* or *countably bounded*. The strictly bounded case corresponds to the commonly used notion of "discrete space" in SA: that is, there is a finite set of spatial units for which we can measure attributes. Countably bounded corresponds to "continuous space" in SA, where any location in the Cartesian space can be associated with a measured attribute. If the number of locations is infinite, it must be countable to support measurement and analysis: that is, the set of locations must have the same cardinality as the set of natural numbers, the smallest infinite set (Beguin and Thisse 1979).[4] This is mostly a theoretical issue with apparently little consequence for practice other than suggesting a limit on SA, since there are sets larger than we could possibly analyze (whether these sets correspond to anything geographical is an open question).[5]

The SA and GIS literature generally assume that the set of locations in the GMF corresponds to a plane. When considering regions approaching the size of continents and above, a spherical GMF can allow more accurate representation of distance, direction, and area. One system for quantifying locations on the curved surface of the Earth is through spherical coordinates (Raskin 1994):

$$\mathbf{x_i} = \left\{ \left(x_i^{\alpha}, x_i^{\beta}\right) \middle| x_i^{\alpha} \in [-\pi/2, \pi/2], x_i^{\beta} \in [0, 2\pi] \right\} \quad (2)$$

Length. The length-metric d_L is a shortest-path relation defined between any two members of X. These shortest paths have the properties of *nonnegativity*, *identity*, *symmetry*, and *triangular inequality*. Nonnegativity means that shortest path lengths are always zero or positive real numbers; identity means that the length between a location and itself is zero; symmetry means that length between two locations is the same in both directions; triangular inequality means that the (direct) length between two locations will be less than or equal to the (indirect) length between the two through a third location. The length-metric is often interpreted as the *distance* between any two locations in the space.

The standard or default assumption in the Euclidean model is that the length-metric is the straight-line segment between any two locations; this corresponds to the Euclidean distance between the pair. Euclidean space is a meaningful framework for representing and analyzing geographic phenomena due to the tradition of projecting the globe to a flat surface as well as to the fact that geographic reality can appear Euclidean (at least to our naked senses). However, geographic processes can have non-Euclidean properties. For example, human-made or natural networks often channel spatial interaction. This means that the shortest path between two locations may no longer correspond to a straight-line segment. If flow congestion is a factor in the channels, the symmetry and triangular-inequality properties also may not hold. There is also strong evidence that individuals' perceptions of geographic space violate the Euclidean space assumptions (Montello 1992).

The following formula can generate length-metrics that obey the nonnegativity, identity, symmetry, and triangular-inequality properties:

$$d_p(\mathbf{x_i}, \mathbf{x_j}) = \left[\sum_{k=1}^{n} \left|x_i^k - x_j^k\right|^p\right]^{\frac{1}{p}}, p \geq 1 \quad (3)$$

where $\mathbf{x}_i$ is a k-dimensional vector of location coordinates and p is a real number referred to as the length-metric parameter. Only one length-metric parameter value ($p = 2$) corresponds to the standard straight-line segment distance assumption. Another important special case is $p = 1$, or the "Manhattan metric," where shortest paths are polylines with segments parallel to one of the axes; this simulates a regular grid network (Shreider 1974; Love, Morris, and Wesolowsky 1988; Puu and Beckmann 1999). Other distance metrics are possible, since interaction along straight-line paths is the exception rather than the rule. For example, distance metrics implied by actual travel distance at urban and regional scales are typically $1 \leq p \leq 2$ (Love, Morris, and Wesolowsky 1988).

Although equation (3) is highly general, it is possible to formulate even more general length relationships, although they will not obey the three metric properties. Smith (1989) formalizes distance as the greatest lower bound on all path lengths between any two locations. The resulting lengths are *quasimetric*, since they only obey the triangular-inequality property. Huriot, Smith, and Thisse (1989) generalize this by formalizing distance as the greatest lower bound of all minimum cost *trips* (possible movements) between any two locations, where "cost" is a very general measure of the difficulty of movement. This conceptualization can accommodate a wide range of possible length measures, including economic, social, psychological, and functional distances. The resulting distance measure is also quasimetric.

The increasing availability of global databases and the rise of global science means that the plane may not be appropriate as a GMF for analyzing some geographic phenomena. Shortest paths within spherical spaces are less well known in the SA (and, indeed, wider geographic) literature, even though they are fundamental to the globe. The shortest path between any two locations on a sphere is the smaller arc of the *great circle* passing through the locations, where a "great circle" is a circle on the surface of a sphere the center of which coincides with the center of the sphere (Shreider 1974; Love, Morris, and Wesolowsky 1988). In a spherical GMF, length is measured through cosines (Raskin 1994):

$$d_{S(r)}(\mathbf{x_i}, \mathbf{x_j}) = r \cos^{-1}(\mathbf{x_i} \bullet \mathbf{x_j}) \quad (4)$$

where $S(r)$ refers to a sphere with radius r and

$$\mathbf{x_i} \bullet \mathbf{x_j} = \sin x_1^{\alpha} \sin x_2^{\alpha} + \cos x_1^{\alpha} \cos x_2^{\alpha} \cos\left(x_1^{\beta} - x_2^{\beta}\right) \quad (5)$$

Spherical length functions obey the metric space properties (nonnegativity, identity, symmetry, and triangle inequality).

Area. An area-measure μ_A is any real-valued function of the locations only (i.e., no attributes) that is additive for disjoint subsets and zero for the empty set.[6] Area-measures are nonnegative, since they are functions of the locations only and not measured attributes. If the area measure for a subset of places is positive and finite, then that subset is *dimensional* (e.g., a line or polygon). If the area measure for a subset of places is zero, than the subset is *adimensional* (i.e., a point).

Although the *topological* or connectivity properties of a GMF are more fundamental than the length-metric, in practice most analysts assume or choose a length-metric that implies a topology. In the Euclidean plane, *neighborhoods*, or the fundamental units of connectivity, are open discs or circles of arbitrary radius without their boundaries (Worboys 1995). A neighborhood is diamond-shaped at $p = 1$ and approaches a square as p approaches infinity (see Love, Morris, and Wesolowsky 1988). The topology determines the collection of subsets of X for which the concept of an area-measure makes sense.[7]

Measuring and Analyzing Non-Euclidean and Nonplanar Spaces

Abandoning the standard Euclidean assumptions requires deciding which GMF is most appropriate for the geographic phenomena being studied. One strategy is to shift the locations in X to the relative locations implied by the geographic phenomenon. This approach changes the locations of X to better fit the phenomena, but retains the standard length and area metric assumptions in the GMF.

Two analytical techniques for inferring the relative locations of places are *multidimensional scaling* (MDS) and *bi-dimensional regression*. MDS attempts to construct a space such that relative locations reflect as closely as possible the space implied by a set of binary comparisons between members of a set (see Golledge and Rayner 1982; Gatrell 1983; Cliff and Haggett 1998). Bi-dimensional regression is an extension of the $\Re \rightarrow \Re$ mapping in standard regression to a $\Re^2 \rightarrow \Re^2$ mapping, where $\Re$ is the set of real numbers (Tobler 1994). Statistical inference tests allow discrimination among hypothesized bi-dimensional regression formats (Nakaya 1997). An especially valuable feature of bi-dimensional regression is output measures, such as displacement vectors, strain tensors, areal distortion, and angular distortion. These can be mapped using GIS to provide powerful visual inferences to the structure of functional and cognitive space.

Another approach to measuring non-Euclidean planar spaces is to retain the absolute locations of X in the GMF and instead estimate the length-metric (and consequent area-measure) implied by the geographic phenomenon. Muller (1982) uses a similar parameter search method to fit a generalization of equation (3). Love and Morris (1972, 1979) estimate the parameters of several metric and semimetric distance functions from observed urban and rural travel distance. These equations can be easily estimated given distance or interaction costs between pairings of locations. If we cannot accept the symmetry assumption inherent in length-metrics, an alternative is to model the resulting space as a vector field in which each location has a direction as well as magnitude (Tobler 1976, 1978; Dorigo and Tobler 1983; Puu and Beckmann 1999).

The estimation and analytical tools described in this section seem to be core spatial analytic tools, since they help define the appropriate GMF for the geographic phenomena being measured and analyzed. As far as we know, no commonly available GIS software supports these estimation and analytical functions. A key requirement is support for a logical data model that can support effectively object pairings and their attributes (see Goodchild 1987, 1998). Most GIS software does not support a simple logical structure, the matrix, which can maintain object pair relationships as well as algebraic manipulation of these quantities (Miller and Shaw 2001). Supporting vector fields requires a raster data structure that can support vectors in addition to scalars, and tools for analyzing and visualizing these vectors. MDS, bi-dimensional regression, and parameter-search-method procedures for estimating distance functions should also be innate GIS tools with supporting user interfaces, visualization and decision support tools, and perhaps intelligent agents, to make these procedures accessible to a wide audience.

Spherical space requires extensive modification of planar space analytical techniques. Three strategies are available for analysis of spherical geometry (Jupp and Mardia 1989; Raskin 1994). The first is to retain planar space and use map projections to transform between the plane and the sphere. This is the implicit approach in the majority of SA, although projection back to the sphere rarely occurs. This approach is problematic, since map projections cannot simultaneously preserve distance and area. A second approach is to represent space intrinsically as non-Euclidean spherical space. This involves analysis using spherical coordinates and trigonometric functions, requiring additional computational overhead to access and execute the trigonometric functions. A third strategy is embedding the sphere within three-dimensional Euclidean space. In this case, we use three-dimensional SA but restrict solutions to the sphere surface. This requires additional data storage overhead for the third dimension, but does not require computationally expensive trigonometric functions.

There are some basic statistical techniques available for the sphere, including interpolation methods, measures of central tendency and dispersion, and hypothesis tests for statistical distributions in spherical space, although these deal mostly with point processes. Correlation measures and regression models are available (Chang 1986), although the regression procedures do not capture spatial autocorrelation among structural variables or error terms. Also required are statistical techniques for line and area processes, as well as a theory and methods of spatial autocorrelation on the sphere (Watson 1983; Renka 1984; Goodchild 1988; Raskin 1994). MDS or bi-dimensional regression methods for spherical space are still required, as is empirical research on estimating spherical distance functions from global-level interaction behavior.

Location analysts have addressed the problem of optimal facility location on a sphere (Drezner and Wesolowsky 1978; Drezner 1981, 1983, 1985; Wesolowsky 1983; Love, Morris, and Wesolowsky 1988; Hansen, Jaumard, and Krau 1994). Tobler (1997) extends the vector field method to spherical space for analyzing interaction at global scales.

Geographic Space

Measuring attributes within the framework imposed by a GMF creates a *geographic space*. From this emerge the properties of geographic relationships and geographic form. In this section, we discuss the measurement and analysis of geographic attributes, the geographic properties implied by these attributes, and the possibilities for analyzing these attributes.

Geographic space consists of the three components of the GMF, along with measured *attributes* (μ_h) from a set of attributes H, corresponding to geographic phenomena:

$$(X, d_L, \mu_A, \{\mu_h : h \in H\}) \tag{6}$$

Beguin and Thisse (1979) note that this formal definition is consistent with Brian Berry's (1964) classic "geographic matrix" characterization of SA. Note that geographic space does not require Euclidean assumptions: the GMF can include any length-metric and area-measure as defined in the second section above. However, most of the techniques discussed in this section assume a Euclidean plane. Extending these techniques to non-Euclidean and/or spherical spaces is an open and worthwhile research frontier.

A *simple attribute* is a measured value defined on a measurable subset of places (i.e., any subset for which an area-measure as defined above makes sense). Two types of simple attributes are *stock attributes* and *flow attributes*. The former is a value assigned to one location. The latter is a value associated with two disjoint places and implies movement of the mass from an "origin" to a "destination." As we noted previously, most GIS software does not support these object pairings and their attributes very well. *Composite attributes* are a function of some combination of a length-metric, an area-measure, and/or simple attributes. These can include geographic densities (i.e., the ratio of a simple attribute to an area-measure, or path distances within a network; see Beguin and Thisse 1979 for details).

Two emergent properties of geographic space are geographic relationships and geographic form. As noted earlier, methods for measuring and analyzing these properties have not spread widely, apparently due to historically scarce data and difficulty in processing the required information, as well as a lack of knowledge about these theories and methods. In this section, we describe the theories and analytical and computational methods for analyzing geographic relationships and form. Since representing geographic attributes on a sphere is different than the plane, we first discuss issues related to representing geographic attributes within a spherical GMF. We then discuss theories and computational methods for analyzing geographic relationships and form.

Representing Attributes in Spherical Geospatial Measurement Frameworks

Similar to planar space, we can represent geographic objects or fields in spherical space using vector-based or tessellation-based data models for implementation within computational platforms. Most vector-based representations in spherical space are straightforward modifications of the planar models. For example, although spherical polygons are comprised of great-circle segments instead of line segments, as in the planar case, they can be stored using the standard endpoint lists, since the underlying geometry is understood to be spherical (Raskin 1994). Available spherical space computational geometry algorithms include methods for determining k^{th} nearest neighbors (Hodgson 1992), computing polygon areas (Kimerling 1984), performing a point-in-polygon test (Bevis and Chatelain 1989), overlay operations (Schettino 1999), and determining the side of an arc where a point lies (Lawson 1984; Renka 1984).

Tessellations of the sphere are more problematic than similar subdivisions of the plane. Unlike the case with planar tessellations such as raster grids, it is difficult to divide the sphere into exhaustive and nonoverlapping regions of equal size and shape: shape, area, or both must be nonuniform (White, Kimerling, and Shar 1998). An alternative approach is to project the sphere onto one of the polyhedral solids, since these can support exhaustive and nonoverlapping regions of uniform size and shape.

Spherical data structures tend to be hierarchical, reflecting the requirement to maintain very large global datasets (Raskin 1994). Tobler and Chen (1986) provide an example of a quadtree structure for maintaining global data. Dutton (1999) develops a quarternary triangulated mesh georeferencing system based on a projection of the sphere to an octahedral solid. Ottoson and Hauska (2002) develop an ellipsoidal quadtree for indexing global geographic data that is based directly on the ellipsoid rather than on an approximation.

Geographic Relationships

Nystuen (1963) identifies three fundamental relationships of geographic space from the spatial analytic perspective: *distance*, *connectivity*, and *direction* (also see Pullar and Egenhofer 1988). In the SA and GIS literature, the measurement of distance and connectivity between spatial objects is often incomplete relative to the range of possibilities. We can also extend the concept of distance to encompass *least-cost paths* in geographic space. Direction has received only limited attention.

Distances between Geographic Entities. A *distribution* of distances, rather than a single distance, exists between two geographic entities if at least one of the entities' geometry is dimensional as defined above (Kuiper and Paelinck 1982; ten Raa 1983; Kuiper 1986). In practice, analysts often use a single, centroid-to-centroid distance measurement as a summary of this distribution. This can be a poor surrogate (Hillsman and Rhoda 1978; Current and Schilling 1987, 1990; Okabe and Miller 1996; Okabe and Tagashira 1996).

Since the distance between dimensional geographic entities is a distribution rather than a single value, we can calculate different measures depending on the application needs. One measure is the *expected distance*. This requires density functions that describe the probability of an interaction between locations in both objects (Larson and Odoni 1981). For tractability, analysts often assume that the interaction probabilities between locations in the two objects are independent—that is, the origin location in one object does not influence the destination location in the second object. Even with that simplifying assumption, only a subset of special geometric cases has been solved, due to analytical intractability (Gaboune, Laportre, and Soumis 1993; Hale 1998).

More tractable is the *average distance*. This is a special case of the expected distance when interactions are equally likely from all locations in both objects (Koshizuka and Kurita 1991). Average distances are useful when measuring interaction "cost" (time, money, energy) between two objects. Average distance is a good summary for total interaction distance: that is, minimizing average distance also minimizes total interaction distance given the equal-probability assumption. The centroid-to-centroid measure discussed above is apparently a surrogate for the average distance, although it is a poor approximation under some circumstances, particularly as the entities are relatively large and proximal in space (Okabe and Miller 1996). As far as we know, all widely available GIS software calculates distances between dimensional entities using the centroid-to-centroid measures, although none bothers to tell users about its potential weakness.

The average distance can be difficult to calculate for arbitrary spatial objects. In two-dimensional Euclidean space, one strategy is to approximate the spatial objects as circular regions (e.g., Rodriguez-Bachiller 1983; Vaughan 1984; Koshizuka and Kurita 1991). Another strategy is to calculate the *root mean squared distance*. This is tractable for a wider range of spatial objects, but systematically overestimates the average distance (see Wilson 1990). Okabe and Miller (1996) develop computational methods for calculating average distance (in Euclidean space) between any pairing of points, polylines, or polygons when stored in the vector spatial data format. Worst-case time complexity is $O(mn)$, where m, n are the number of line segments in the polyline or polygon boundary of each object.[8] This limits the application to objects where $n, m < 1000$.

Another possible distance measure is the *minimum distance* between two objects. This is appropriate when the interaction is affected by proximity. Minimum distance calculation is closely related to a well-studied problem in the computer-science literature: namely, the *closest-pair problem* for a finite set of points. Peuquet (1992) develops a quadtree-based algorithm for calculating the minimum distance between arbitrary and disjoint spatial objects. Fujishige and Zhan (1992) specify a method for two polytopes. Okabe and Miller (1996) develop minimum distance methods based on Voronoi diagrams. Worst-case time complexity is $O(n)$ for point-line and point-area where the line and area are described by an n segment polyline or polygon and $O(m+n)$ for line-line, line-polygon, and polygon-polygon pairings described by polylines with m and n segments respectively. This is tractable even for large problems.

Two types of maximum distances between spatial objects are possible. First is the distance between the two farthest locations. Okabe and Miller (1996) also develop methods for maximum distance calculations between points, lines, and polygons. These methods are variants of the minimum distance calculations that involve *farthest-point Voronoi diagrams.* These methods require only linear time in the worst case, meaning that they are tractable even for very large datasets. Another possible maximum distance is the *Hausdorff*, or "maximin," distance. This is the maximum of the distances between locations in one object and the closest location in the other (Preparata and Shamos 1985). Note that the Hausdorff distance is not symmetric. Atallah (1983) presents a linear-time algorithm for the Hausdorff distance between convex polygons.

Perception of distances between geographic features is often considerably less precise than the quantitative relationships described above. Humans acquire knowledge about distance based on environmental features encountered during movement. This is mitigated by attention-related factors, such as environmental familiarity and travel purpose (see Golledge and Stimson 1997; Montello 1997). The contingent and inexact nature of this process suggests that qualitative distance relationships (e.g., "closer," "farther") are perhaps more meaningful than distance measures in structuring human spatial behavior. Gahegan (1995) discusses the use of "semiquantitative" linguistic operators and fuzzy-set membership functions to reason about proximity relationships. These operators could be used with *spatial reasoning systems* that infer geographic information from imprecise, incomplete, and subjective descriptions (e.g., Frank 1992; Jungert 1992).

Least-Cost Paths through Geographic Space. The central property of distance from the GMF can also be extended to the concept of least-cost paths through geographic space. The distinction is that in geographic space, an assumption is made that one or more attributes of the geographic space affect movement or interaction; examples include land cover, terrain, and traffic congestion. Least-cost paths through geographic space require one or more measured attributes to be interpretable as an interaction cost. We refer to this structure as a *cost-density field*. A cost-density field is a function distributed continuously on the plane with no points of concentration (Beguin and Thisse 1979). Conceptually, the cost-density field defines how much resource (energy, money, time) is required for moving across locations in the plane.

Given a cost-density field, we can determine the least-cost path between any two locations by solving a classical problem from the calculus of variations that minimizes the cumulative cost of a continuous path between a location pair (Warntz 1965; Wardrop 1969; Angel and Hyman 1976; Puu and Beckmann 1999). Direction-specific cost can also be accommodated (see Puu and Beckmann 1999). Solving the variational problem for the general case is difficult. Analytical solutions usually require restrictive assumptions, such as the *radially symmetric* case, in which cost is a function of distance from a single point location such as an urban center (see Angel and Hyman 1976). Although analytical solutions to the continuous space problem are difficult, there are tractable computational solutions for several discrete approximations. However, there are no widely available software tools, nor are these techniques widely applied in SA or GIS. The discussion below is based on (but also expands on) Miller and Shaw (2001, chapter 5).

Special case 1: Cost polygons. The "shortest path through geographic space" problem is more tractable if a tessellation of cost polygons represents the cost surface. In this case, the problem of finding the minimum cost path reduces to finding the locations where the path crosses the boundaries, since the path within a polygon is a line segment. Werner (1968) demonstrates that the boundary-point location problem can be solved using the *law of transportation refraction*. This reformulation of Snell's law for the refraction of light provides the first-order optimality conditions for the boundary point (see Werner 1968, 1985 for more detail).[9]

Smith, Peng, and Gahinet (1989) and Mitchell and Papadimitriou (1991) formulate scalable procedures for the case where the tessellation is a triangulation of the two-dimensional plane. Smith, Peng, and Gahient (1989) use a generalization of Snell's Law to construct a family of local, asynchronous, and parallel algorithms that find least-cost paths when the cost polygons are a triangulation. *Local* means that each processor only requires information from neighboring processors. *Asynchronous* means that neighboring processors do not execute simultaneously. These properties are achieved by restricting crossings to finite locations within the boundary segments of a triangle. The algorithms are exact for the special case of a corridor of triangles along a single dimension. Globally optimal paths are not guaranteed for the two-dimensional triangulation, but numerical experiments are promising.

Mitchell and Papadimitriou (1991) apply Snell's Law for the case where the tessellation is a constrained Delaunay triangulation. Their procedure generates a "map" of shortest paths from a given source location, allowing the user to determine a specific shortest path by querying from the database. The procedure requires $O(n^4)$ time in the worst case but is likely to be shorter in practice.

Another subcase occurs when the polygons correspond explicitly to a polyhedral terrain in three-dimensional Euclidean space. In this case, the polygons are not weighted by cost; instead, the polyhedral surface is a distortion of Euclidean space, and the objective is to find the minimum Euclidean distance path on this surface. Mitchell, Mount, and Papadimitriou (1987) develop an $O(n^2 \log n)$ procedure for arbitrary (possibly nonconvex) polyhedral surfaces. De Berg and Van Kreveld (1997) develop procedures that determine shortest paths over polyhedral surfaces that are restricted to stay below a specified elevation or minimize total ascent. This is useful for cross-country movement-planning in mountainous terrain.

The *shortest path through cost polygons* problem is a particularly vivid example of the theme of this article. This idea has its roots in the early SA literature. Once processors were fast enough, GIScientists and computer scientists exploited this idea to build computationally scalable routing methods. But no commercial GIS software implements these tools at present, and to date, they have not been widely applied in SA or integrated into SA techniques that require distance or minimal cost path measures.

Special case 2: Finite locations. Another approximation is to restrict the set of locations to a finite set in n-dimensional space. We only define interaction cost for pairings of these locations. This defines a *network* and allows us to solve the approximation as the more tractable *shortest path in a network* problem. Conversion to network representations is a form of controlled relaxation of the Euclidean space assumptions used by spatial and other analysts for decades (Tobler 1993), although these representations often correspond to physical networks in the real world.

Network-based approaches can be combined with special-case 1 to simultaneously model within-network and cross-country movement. If we constrain the Delaunay triangulation boundaries to match road features in the landscape, within-network flow can be modeled using the resulting triangulated irregular network (TIN) structure, while cross-country movement can be modeled using the Mitchell and Papadimitriou (1991) algorithm (van Bemmelen et al. 1993).

Special case 3: Cost lattice. Another tractable approximation is to restrict the locations to a discrete and finite *lattice* of the plane and specify interaction costs only between neighbors. We can define the neighbors based on the *rook's case* (proximal locations only in directions parallel to the axes), the *queen's case* (rook's case plus proximal locations along the diagonals), or the *knight's case* (the queen's eight moves plus "L" shaped moves). This is still a network representation, but the regularity and density of locations can better represent the properties of some continuous surfaces, such as terrain or land cover. We typically interpret each lattice point as the centroid of a small raster cell exhibiting that interaction cost value.

The lattice approximation introduces three types of error: *elongation*, *deviation*, and *proximity* distortions (Goodchild 1977). Elongation errors occur, since the lattice path will be longer than the continuous space path. Deviation errors occur when the lattice path differs in location from the continuous space path; this is maximal when all moves in one direction are executed first. Proximity distortions occur when the cost measure for a raster cell does not consider neighboring cells. The resulting paths can be optimal with respect to their site but suboptimal with respect to their situation (Goodchild 1977; Huber and Church 1985; Lombard and Church 1993; van Bemmelen et al. 1993).

Intuition might suggest that as the lattice becomes denser and closer to continuous space, the lattice solution approaches the continuous-space solution. However, elongation and deviation errors are independent of the lattice density. Instead, these errors relate to the permutation of move directions in the lattice-based path. These cannot be eliminated through a finer mesh (Goodchild 1977). Other strategies for reducing these errors include *more connected rasters* and an *extended raster approach*. The more-connected raster approach connects each lattice point to more of its proximal neighbors in Euclidean space (e.g., connect to the closest 16, 32, 64, . . . lattice points instead of just four or eight, in the rook's and queen's cases, respectively). This can reduce elongation and deviation errors, but requires additional computational expense and can create nonintuitive intersecting paths. The extended raster approach configures the network so that a path enters and exits each cell at specified locations at the boundaries, rather than traveling between the centroids of each raster cell. The cost of each segment of the path is easier to calculate, since it is contained within a single cell. These strategies can also be combined with a quadtree structure for reducing data storage and computational requirements (van Bemmelen et al. 1993).

There are generally few tools or applications of shortest paths through geographic space in widely available GIS software, although the related problem of hydrological network extraction from digital terrain models is well studied (see Band 1999) and more widely available. A general modeling system and software toolkit by Burrough (1998) analyzes the distribution and transportation of material over a raster structure. A wide range of dynamic geographic processes can be captured, including deterministic and stochastic processes and processes without and with memory (an example of the latter case is the flow of water through a capacitated medium over time). Since the system can be described and implemented as local operations within a raster tessellation, it is computationally scalable to very large dynamic geographic processes. However, as Malanson and Armstrong (1997) demonstrate, these modeling systems are sensitive to modifiable areal unit effects related to the total gradient represented, number of raster cells, and step size between cells.

Connectivity between Geographic Entities. In SA, the concept of "connectivity" is often reduced to a relatively simple binary condition or intensity measure. For example, contingency matrices often record a connectivity relationship such as nearest neighbor, adjacency in a graph, or a shared boundary as a binary variable. These variables can also be weighted—that is, a function of the nearest neighbor distance, arc length, or length of the shared boundary. This is a limited view of the possible topological relationships between two geographic features when one or both is dimensional.

Using point-set topology as a foundation, Egenhofer and Franzosa (1991) and Egenhofer and Herring (1994) substantially expand conceptualization and representation of topological relationships between spatial objects in $\Re^2$. The *4-intersection model* identifies possible topological relationships between spatial regions[10] based on the set intersections between their boundaries and interiors (Egenhofer and Franzosa 1991). The *9-intersection model* extends the 4-intersection model to encompass topological relations involving lines by considering the set intersections between the object's interiors, boundaries, and exteriors (Egenhofer and Herring 1994). The 9-intersection approach can also be embedded within a temporal framework (Egenhofer and Al-Taha 1992).

The 9-intersection model illustrates the diversity of potential connectivity relationships between geographic features. For example, there are thirty-three possible topological relationships between two simple lines, with an additional twenty-four relationships possible if the lines are not simple.[11] Twenty possible topological relationships exist between a line and a region, and eight possible topological relationships exist between two regions. Human-subjects experiments suggest that the line-region relationships correspond to natural language descriptions of real-world geographic features by human subjects (Mark and Egenhofer 1994; Knauff, Rauh, and Renz 1997).

The "Egenhofer relations" only require testing for the existence of intersections among the boundaries, interiors, and exteriors of spatial objects. These tests are tractable, subject to the accuracy of the input data and precision of the computational platform (see Preparata and Shamos 1985; Worboys 1995). These tools are not evident in commonly available GIS software. However, if spatial database-management systems incorporate Egenhofer relations for spatial querying, users could write their own code in embedded query languages, such as SQL3, that support user-defined objects and relationships

Directional Relations between Geographic Entities. Directional relationships between geographic entities have received limited attention in SA and GIS, although this is changing (e.g., Lee and Wong 2001). The importance of directional relationships is well recognized in physical geography with regard to issues such as capturing the interactions of material or energy due to wind or water. For example, wind direction is critical when analyzing and predicting the dispersion of air-pollution plumes from point sources and/or nonpoint sources (Hruba et al. 2001; Loibl and Orthofer 2001). Directional relationships are less commonly considered in human geography, although they can be important in human spatial perception and behavior. For example, individuals can have directional biases in their knowledge of and movement within an urban area (Adams 1969; Moore 1970; Moore and Brown 1970). At regional and continental scales, individuals exhibit clear biases towards the four cardinal directions in their mental images of spatial relationships (Mark 1992; Egenhofer and Mark 1995).

Both quantitative and qualitative procedures exist for analyzing directional relationships. A quantitative theory is available for point objects; theories of directional relationships among dimensional spatial objects are qualitative. A quantitative theory of directional relationships among dimensional objects is an open and valuable research frontier.

Directional statistics are a suite of techniques for statistical inferences from directional observations of point processes. Directional observations are typically made from a reference location in $\Re^2$. These observed directions are generally treated as realizations of a random process on the unit circle centered on the observation point. (Spherical statistics, discussed previously, are a special case of directional statistics defined on the unit sphere.) Measures of central tendency, dispersion, goodness of fit to theoretical distributions, and sample difference tests are available (Mardia 1972). Klink (1998) illustrates the use of directional statistics in conjunction with scalar and vector-based methods for analyzing wind fields and inferring the factors that influence these processes. As with spherical statistics, a theory of directional statistics for line and area processes is still required.

Directional autocorrelation statistics are an extension of spatial autocorrelation statistics from one-dimension (distance) to two dimensions (distance and direction). Directional autocorrelation statistics have been used to study distributions of genetic structure, anthropological features (e.g., cranium measurements), and cancer mortality over space (see Rosenberg 2000). However, many directional autocorrelation methods are ad hoc extensions of standard spatial autocorrelation statistics. For example, the "windrose" approach of Oden and Sokal (1986) aggregates observations into independent distance/direction classes, each having its own weight matrix. This method requires large sample sizes, since each distance/direction class must have a sufficient number of observations. Not very satisfactory solutions to this sample-size problem include weighting distances with the angle relative to the reference location (Falsetti and Sokal 1993) or projecting the locational coordinates onto a fixed bearing with respect to the reference location. Rosenberg (2000) develops a bearing autocorrelation statistic that improves these methods by formulating a nonbinary spatial weights matrix that simultaneously reflects distance and direction. This results in a bearing autocorrelation statistic that is more directly comparable to traditional spatial autocorrelation statistics.

Computational procedures are available for assessing and reasoning about qualitative directional relationships among dimensional geographic objects. Peuquet and Zhang (1987) develop a linear-time algorithm to determine directional relationships between arbitrary polygons in $\Re^2$. They use a cone-based search strategy that determines directional relationships relative to the eight angular regions defined by the four cardinal directions (e.g., north, south) and the four diagonals (e.g., northeast, northwest). Papadias and Egenhofer (1996) use a hierarchical method that generates projection lines perpendicular to the coordinate axes. This determines directions based on rectangular subregions of $\Re^2$ relative to a reference location.

Geometric Form

Geometric form can provide insights into geographic phenomena, particularly when linked with process theory (King 1969). Properties such as shape, configuration, and pattern preserve information on the processes that influence the entity's development (Bunge 1966; Whyte 1968). Comparison among geographic forms can highlight differences in the development paths of the entities (Tobler 1978). Geographic form can also guide the search for process explanations (Lo 1980; Dobson 1992). Analyzing the mismatch between predicted and observed form can provide insight (Simons 1973–1974, 1974; Eason 1992; Batty and Longley 1994). Geometric form also influences the functioning and growth of geographic entities. Boundaries can dampen or enhance interaction across and within boundaries (Longley et al. 1992; Haines-Young and Chopping 1996; Nystuen 1997). Shape and pattern also affect human processes such as spatial behavior and wayfinding by affecting environmental legibility and recall (Gluck 1991; Miller 1992).

Shape analysis is a body of techniques for analyzing geometric form. No commonly available GIS software has an adequate set of shape-analysis tools. This is probably due to traditional difficulties in conceptualizing and measuring shape as well as computing the required geometric properties. However, there has been recent progress in shape analysis that, combined with faster processing speeds and more detailed digital geographic data, can make these tools more useful in SA.

A common conceptualization of shape in the SA and geographic literature involves the dimensions of *compactness*, *concavity*, and *ellipticity* (Austin 1984). This conceptualization seems driven more by convenience than theory, since properties are easily isolated and measured using traditional (in particular, analog) methods. A more defensible conceptualization involves the dimensions of *edge roughness*, *perforation*, and *elongation*; these follow from the mathematical definition of shape as the residual properties after removing translation, rotation, and dilation transformations (Wentz 2000).

Another problem with shape analysis is difficulty in capturing the conceptualized shape dimensions in a manner that preserves information—that is, one that can discriminate among all possible shapes. A common approach is to compare the perimeter, area, or distances within the spatial object to an idealized reference object, such as a circle. These methods can capture only one shape dimension, such as compactness or ellipticity (Massam and Goodchild 1971; Massam 1975; Austin 1984). Fractal-based measures capture only boundary roughness (see Lam and DeCola 1993). The Boyce-Clark (1964) radial vector measure is more comprehensive, but is sensitive to the number of radii and the object's orientation. Medda, Nijkamp, and Rietveld (1998) eliminate these arbitrary aspects of the Boyce-Clark method. The dual-axis Fourier shape analysis (DAFSA) is information-preserving, in the sense that the object can be recovered from its parameters (Moellering and Rayner 1981), although it may capture arbitrary (nonshape) information about the object (Griffith 1982). The trivariate shape measure has separate indices for the dimensions of edge roughness, perforation, and elongation, but is interpreted jointly (Wentz 2000).

A third problem is that many shape measures are not computationally efficient and therefore do not scale to handle very large databases. Methods that compare test objects to reference objects are not computationally burdensome, but also do not tell much about shape. The Boyce-Clark method is more complete, but has a higher computational burden, particularly for Medda, Nijkamp, and Rietveld's (1998) enhancement. Fractal measures are scalable computationally, but convey information only about roughness properties. The trivariate measure uses a fractal dimension calculation combined with other scalable measures to convey a wide spectrum of unambiguous shape information (Wentz 2000).

Some computationally scalable procedures are available for making comparisons among forms. Bi-dimensional regression and other statistical techniques can compare among forms (Tobler 1978; Bookstein and Sampson 1990). Arkin and colleagues (1991) use angular calculations to compare polygons represented using the vector GIS format (i.e., as a closed polyline). Some techniques also exist for shape analysis of imagery data, including biomedical images (e.g., Banerjee and Dutta Majumdar 1996) and fractal methods for remotely sensed data (e.g., DeCola 1989; Lam 1990; see Wentz 2000).

There are also some techniques for evaluating geometric form based on computable models of geographic fields, such as lattices and TINs. These include methods for calculating simple properties such as slope and aspect, including error estimates and sensitivity analyses (e.g., Chang and Tsai 1991; Hodgson and Gaile 1996; Hunter and Goodchild 1997). Also available are methods for calculating fractal dimensions of terrain features (e.g., Burrough 1981; Clarke and Schweizer 1991; Helmlinger, Kumar, and Foufoula-Georgiou 1993).

Connecting shape to process can benefit from a language to describe the evolution of shape over time. Stiny (1980) develops a *shape grammar* that consists of a set of shapes, a set of symbols, a set of shape-transformation rules, and an initial shape. This system can support rigorous analysis of shape evolution and its influence on geographic processes. Krishnamurti and Earl (1992) extend this approach to three-dimensional space. Chase (1997) combines this approach with predicate logic for inferring high-level geographic relationships in digital geographic databases.

Determining relationships between geographic process and geographic form must proceed with caution. Geographic fields and objects are, to some degree, artifacts of the measurement process. Since field and object-based representations are inverse constructs, they are arbitrary, although one may seem more natural than the other for a given application (Couclelis 1992; Worboys 1995). Also, the crisp representation of an object is often counter to the geographic entity's inhomogeneous nature and a lack of well-defined boundaries. Factors such as scale, resolution, perspective, and the user's purpose can dictate an object-based representation (Couclelis 1996; Plewe 1997; Tate and Atkinson 2001). Boundaries can be arbitrary and create yet another manifestation of the modifiable areal unit problem by artificially bounding geographic processes (Griffith 1982). Aggregating elemental spatial units can create zones with shape and boundary effects that were not present in the underlying geographic process, requiring sophisticated analytical tools to compensate (e.g., Ferguson and Kanaroglou 1998). Similarly, scale and resolution affect pattern measurement (Turner, O'Neill, and Gardner 1989; Moody and Woodcock 1995).

Other problems when connecting form to process relate to the nature of nonlinear processes. As mentioned previously, nonlinear processes can be sensitive to initial conditions, generating divergent geographic forms from very similar initial configurations. Also possible is equifinality, or diverse processes converging to the same macrostructure. It can be difficult to separate deterministic (and therefore reducible) uncertainty from purely random (and therefore irreducible) components in spatiotemporal processes (see Phillips 1997, 1999).

GIS has great potential for resolving some problems associated with linking process and form. GIS can support sensitivity analysis based on variable representations (Fotheringham and Rogerson 1993). Also emerging for dealing with ambiguous or imperfectly known boundaries are fuzzy-set-theory shape techniques (see Chaudhuri 1990) and shape-reasoning systems (see Schlieder 1996).

Conclusion

The Beguin-Thisse theory of analytical geographic spaces provides a useful focus for considering representational and analytical alternatives to the Euclidean model. This theory suggests that underlying geographic attributes is a geographic measurement framework (GMF) that strongly influences measurement by specifying the set of locations, shortest path distance relations, and measurement of area-based properties. The GMF must be carefully considered and possibly estimated rather than accepted.

Measuring attributes within a GMF allows analysts to conceptualize and measure geographic objects and relationships among these objects. Key geographic relationships are distance, connectivity, and direction. Distances between dimensional geographic objects are often incorrectly measured in many SA techniques and GIS software; some efficient techniques for appropriate distance measurement exist, as do needs for developing and benchmarking some heuristics. It is also possible to generalize the distance relation to a least-cost path through a cost field, such as terrain or land cover. There are several tractable methods for finding these paths based on discrete-space approximations. Connectivity relations in the SA literature are less sophisticated than those conceptualized and analyzed in the GIScience literature. A small but growing number of direction-based statistical and computational tools exists. These should be expanded and integrated into other SA techniques, such as autocorrelation and interaction modeling. Digital geographic data allow detailed representation of geographic objects; this information can be exploited using new shape-analytic techniques and grammars.

There are several research and development frontiers for alternative geographic representation in SA and GIS tools. A broad-ranging research agenda involves determining the geographic phenomena and problems that benefit from alternative representations and their analytical possibilities. We have made several tentative suggestions in this regard. There are also specific research and development frontiers, many of which serve this overall goal. These include:

1. GIS software tools for estimating and testing GMFs, including multidimensional scaling, bi-dimensional regression, and distance metric parameter estimation techniques.
2. Additional techniques for spherical SA with corresponding GIS software tools and spherical spatial data models.
3. GIS tools that appropriately measure distance between dimensional geographic objects. There is also a need for developing and benchmarking heuristics for average and expected distances.
4. Techniques and corresponding software tools for analyzing relationships based on least-cost paths through geographic space, included those embedded within non-Euclidean and/or spherical spaces.
5. Analytical techniques that capture the full spectrum of possible connectivity relationships between dimensional geographic objects.
6. Analytical techniques and GIS software tools for analyzing directional relationships between geographic objects, including those embedded within non-Euclidean and/or spherical spaces.
7. A quantitative theory and methods for directional relationships among dimensional geographic objects.
8. Additional techniques and GIS software tools for shape analysis, including those for objects embedded in non-Euclidean and/or spherical spaces.

Required enabling technologies include GIS and related software tools that support the representations and analyses discussed in this article. It is unlikely that the GIS software vendor community will provide these tools directly, at least initially, due to lack of market support. Even if the vendor community is responsive to this call, however, the value of these tools will be blunted if they are proprietary. The research frontiers identified in this article involve the integration of detailed georepresentation with SA and detailed geometry with process. The technologies to support this integration must be interoperable, supporting common standards and protocols that allow integration across heterogeneous software environments. Scalability to large geodatabases with complex spatial objects and relations will most likely require efficient software for spatial database management, spatial querying, and geovisualization combined with customized code for SA and modeling, written in high-performance programming languages rather than computationally cumbersome macrolanguages. There is also a need for

SA software clearinghouses where software tools can be shared among members of the research and practitioner communities. These clearinghouses could reside in university settings, perhaps supported by both public and private sectors.

Requirements for interoperable, computationally scalable, and extensible software tools suggest the need to develop open software standards such as the OpenGIS Consortium. The current OpenGIS data model supports points, polylines, and polygons, as well as collections of these objects. However, the data model does not support field-based approaches to georepresentation, which are required for techniques such as shortest paths through geographic space (see above). There is also support for some distance and topological relationships, but the currently implemented relationships are limited with respect to the spectrum outlined in this article. A worthwhile research and development topic is extension of the current OpenGIS data model to include the full range of geographic entities and relationships identified in this article.

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Notes

1. "Analysis" is a multifaceted term. We refer here to its precise mathematical definition as the class of sciences that examines exact relations between quantities or magnitudes (*Webster's Unabridged Dictionary* 1998). Analysis can only examine real-world properties that are measurable, countable, or formally comparable. This definition is consistent with the spatial analytic tradition in modern geography (Taaffe 1974). While there are other valid forms of inquiry, including narratives and graphics, these are not the direct concern of the discussion in this article.
2. Following standard usage, we use the term "geographic information systems" (GIS) to refer to the technology and "geographic information science" (GIScience) to refer to the theories and methods that underlie the technological implementation.
3. We purposely avoid naming any commercial or public-license GIS software directly. When we use the phrase "most GIS software" (or a variation), we imply precisely that: there is one or a very small number of software packages to which this observation does not apply. We decline to name the specific GIS software, either in a positive or a negative light.
4. "Countable" means that we can derive a one-to-one correspondence between the set in question and the set of natural numbers. In 1873, Georg Cantor proved that different "sizes" of infinite sets exist, with the natural numbers being the smallest. The set of irrational numbers is a larger and uncountable infinite set (Borowski and Borwein 1991). The term "σ-bounded" is often used to designate "countably bounded."
5. See Flake (2000) for an excellent discussion of the relationships between computability and natural (including human-made) systems.
6. More precisely, this set is the *Borel σ-algebra* associated with (X, d_L). This is the smallest σ-algebra that contains the open subsets defined by (X, d_L). The σ-algebra of a set is a collection of subsets that contains: (1) the set itself; (2) the empty set; (3) the complements of all members of the set; and (4) all countable unions of members of the set (Borowski and Borwein 1991; see Beguin and Thisse 1979 for an alternative definition). Any Borel set is *measurable*, meaning that we can define a "measure" as indicated in the main text (see Haaser and Sullivan 1971).
7. Note that the concept of "area" does not require a metric space topology; see Casati, Smith, and Varzi (1998).
8. $O(\ )$ or "big oh" notation indicates the order or general complexity class of the algorithm in the worst case. For example, $O(n^2)$ states that the algorithm will require no more than n^2 operations, subject to a proportionality constant. For more information on complexity analysis, see Garey and Johnston (1979) and Sipser (1997).
9. The law of transportation refraction assumes a cost-density field with no points of concentration or other impenetrable objects that can "block" shortest paths. Incorporating these objects requires an analogous concept of diffusion. Thanks to Mike Goodchild for pointing this out.
10. Or, more precisely, *cells*. A cell is a connected two-dimensional region with no "holes." See Worboys (1995) for a more precise definition.
11. A *simple line* is a one-dimensional spatial object that is topologically equivalent to a straight line (Worboys 1995): in other words, the line does not "cross" itself.

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